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This research effort was directed toward offering a practical, effective guidebook for the application of statistical trend analysis methods. Standardization/Evaluation offices throughout the Air Force are required to do trend analysis on measurements taken over time, but those doing it often have no mathematical background. Chapter IV, the user's guide, is written, for them. Ten trend description/trend evaluation methods are offered, nine dealing directly with trend analysis and one with (Cont'd)

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# A GUIDE FOR THE APPLICATION OF TREND ANALYSIS AND OTHER STATISTICAL METHODS

THESIS


AFIT/GST/SM/79M-1 David A. Brunstetter
Major USAF


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by

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## Preface

The research and writing that went into this thesis was performed to assist those who are required to perform trend analysis. If any of the readers of the guide (Chapter IV) find it a useful tool, my efforts will have been worthwhile.

The guide portion of this thesis was written with great detail and extensive explanations. This made it quite long, but hopefully the guide itself can be extracted from the main body of the thesis so it will not look too unwieldy. Also, some parts may seem to be over-explained or repetitious. Hopefully the user will not feel his intelligence is insulted but realize I had to avoid inadequate explanations. The guide is intended for those with very little mathematical background.

I wish to express my gratitude to the original suggestor, Major Michael Dumiak, and Colonel Ronald Luhks, who offered it as a thesis topic. Both were very helpful in getting the research started. Also, I want to thank my advisor, Major Saul Young, because sessions with him tended to ease frustration and put me more at ease so I could write more productively.

Finally, my most heartfelt appreciation and love are extended to my wife, Sandy, for her understanding, personal sacrifices, and moral support gladly given.

David A. Brunstetter

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## Abstract

This research effort was directed toward offering a practical, effective guidebook for the application of statistical trend analysis methods. Standardization/ Evaluation offices throughout the Air Force are required to do trend analysis on measurements taken over time, but those doing it often have no mathematical background. Chapter IV, The User's Guide, is written for them. Ten trend description/trend evaluation methods are offered, nine dealing directly with trend analysis and one with correlation. They are discussed in detail, then are presented in step-by-step guidesheet form. The significance of trends are evaluated using the z-statistic for large samples and the t-statistic for small samples from normal populations. Three more easily applied non-parametric methods are offered for use when time is limited or assumptions for other methods cannot be met.

# A GUIDE FOR THE APPLICATION OF TREND ANALYSIS AND OTHER STATISTICAL METHODS

## I. Introduction

### Statement of the Problem

Management directives throughout the operational commands require staff agencies to monitor training program effectiveness through trend analysis. Unfortunately, little or no guidance is given as to how to measure, describe, or even recognize such trends. In fact, quite often higher levels of management do not define what their view of trend analysis is, or what they expect of it. Even when the concept is adequately described, those made responsible for its use are not chosen for their mathematical ability or background, but their expertise in the specific type of performance that is being measured. That is, those involved in trend analysis need a primer on statistical methods, especially when it comes to declaring when a trend is significant in a statistical sense and when inferences can or cannot be made. The expertise should be applied to determining <u>why</u> there is a trend, not in trying to determine if one exists or not.

## Objective

The ultimate objective of the thesis is to write a guide or primer that will assist those responsible for trend analysis and related statistical methods. This guide should be flexible enough to be used widely, but not so general that its generality detracts from its usefulness. It should also be written in a step-by-step manner easily understood by the unsophisticated (in math) but at the same time well substantiated by references and discussion in the text. This substantiation should be put in a logical manner for the user so that he might have some confidence in the methods, if only from a conceptual point of view.

## Background

An aircrew member at a base in Germany originally suggested a guide of this sort through the Air Force Suggestion Program. As wing radar officer, one of his responsibilities was the development and maintenance of a trend analysis (TA) program. He did not have a lot of time to spend at it, since TA was not his primary job. Furthermore, since he didn't know much about what TA was, (History major) he was spending more time than he could afford. The suggestion was that a publication or manual be written on what TA was and "how to do it". It would save many man-hours, since so many flying organizations had such an additional duty for someone. The suggestion came through channels and ended up as a possible thesis topic.

Subsequent conversations between the suggestor and the writer allowed the general topic to be narrowed down somewhat. One of the most common requests by commanders was for TA officers to be able to say when changes in aircrew performance scores (such as ratio of targets acquired to targets attempted on radar) became significant. Also of interest was the amount of correlation between failures to acquire and weather factors, equipment malfunctions, and so on. It was mentioned that while there was a USAFE Regulation requiring TA, there was no guidance about how to develop such a program (Ref 9:1-2).

## Scope

I will limit my coverage to types of trend analysis and statistical methods mentioned by the original requestor and those I have seen used in my own operational experience as aircrew member and evaluator. This will include various types of time-series, regression, and correlation analysis, plus non-parametric methods and a thorough discussion of statistical significance for each method.

## Anticipated Problems and Limitations

A combination of the above mentioned conversation, subsequent ones, and preliminary research of Air Force manuals pointed out a possible problem. A statistical treatment was lacking and needed, but more fundamental was the lack of proper guidance. The writer decided to do a series of interviews with those involved in TA to get an

idea about how widespread this facet of the problem was.

Two weeks were set aside for this series of telephone interviews. Besides the motivation mentioned above, the writer felt these conversations would be a last chance to find out that the problem was isolated (not worth a thesis effort). Also, they could be a source for problem types, data, and references.

Face-to-face interviews would have been more useful, but there was no time for that. Although the interviews were kept unstructured in order to keep them informal, answers to three specific questions were sought from each interviewee. Guidance was never mentioned in a question, since I wished to see if the interviewee would mention lack of guidance on his own.

The first question was what their own definition of TA was. There were answers all the way from strictly "records-keeping" to simply identification of this month's weak area, all the way to a comparison over time to a set air-crew performance standard. In this last office, the officer mentioned that when performance measures dropped by 3%, it was identified as a trend.

The next question was about whether each felt what he was doing appropriate or not. Nearly everyone interviewed was a little unsure as to the value of what they were doing. There were complaints that commanders tended to "overreact" to what they felt were insignificant trends, but they had no real basis on which to argue. One officer said he was

fairly happy with his program even though he throws away all past data except the current time-period and one time-period back. He said he did this because the rules changed so often.

Lastly, they were asked about what they would want a guide to do for them if they could have one made-to-order. Number one priority was a good definition, some guidance about what TA was and what was expected of it. Number two was something on inferences, about when a trend or change becomes significant. Number three was something on sample sizes and what sample sizes were reasonable when making inferences.

This discussion of anticipated problems (suggestor) and confirmed problems (interviews) leads directly to the limitations of this research effort. This paper will not address the lack of guidance issue as such, it will have to be worked out between the user and his boss. It is felt, however, with the statistical definition of TA which will be addressed, the user will be better equipped to come to an agreement, or at least some common ground upon which to argue.

This paper will address the statistical side of the question and provide a tool for the user along with some idea about what the methods can and cannot do.

## Methodology

In addition to the literature search of Air Force publications, a DDC and Rand bibliography search was

conducted. Business publication indexes were also reviewed since I was interested not only in whether the military already had such a guide, but whether there was something in the civilian sector. None of these sources treated TA the way I felt it needed to be treated. As I viewed it, the TA officer is there as an expert crew-member, not an expert mathematician, and may not understand a sophisticated treatment. What he needs, then, is a translation of these sophisticated methods into something easy to apply. A wealth of material was found in various professors' libraries, the Engineering School library, the Logistics School library, and the Wright State University library. It was determined that the methods found in these statistics and business texts were applicable, particularly those dealing with time-series, straight-line or curve-fitting and correlation. Those statistics texts written for business students were particularly useful, and are excellent references for the user of the guide.

A sufficient number of different methods are discussed in preliminary chapters to give the user a feel for each method if he has the time and the inclination to read them. Also, as much discussion of statistical inference as possible due to space and time is given. The guide itself, however, should stand alone and be usable to one who has not seen the remainder of the paper. Also, each method is self-contained with a minimum of references to other methods.

An example is given for each method, following each step from the guide. Assumptions necessary for each method precede the first step. Cautions and notes on application follow the last step so there is less chance of mistakes. The bibliography is divided up into those references used for method translation and those that are especially useful to the user of the guide.

## II. Tests of Hypotheses

### Levels of Measurement

Measurement is the process of assigning numbers to objects or observations. The kind of measurement which is achieved is a function of the rules under which the numbers were assigned. The operations and relations employed in obtaining the scores define and limit the manipulations and operations which are permissible in handling the scores; the manipulations and operations must be those of the numerical structure to which the measurement is compatible (Ref 6:29).

Nominal. The nominal, name only, or classificatory scale of measurement is measurement at it's weakest level. Numbers may be assigned to these classes or categories, but they serve as nothing more than symbols. For example, if there are only two categories the measurements may fall into, the measurements or "responses" may be symbolized by yes/no, plus/minus, $3.7/\sqrt{3}$, etc. Aircraft tail numbers are nominal, as long as the effect of when or where they were produced is negligible. Due to the nature of nominal data, the only descriptive statistics that may be used are frequency counts and the mode. Tests of hypotheses (covered in the next section), regarding the distribution of cases among categories can be accomplished using some non-parametric methods, and those based on the binomial expansion (Ref 6:23). These tests are appropriate because they

focus on the number of measurements in each category, no matter what the "name" of the category itself is. The mean and standard deviation cannot be generated for nominal data. For instance, it does not make sense to say what is the average (mean) of a yes and two no answers.

Ordinal. This higher level of measurement implies that one category is not only different from all other categories, but that they can be arranged or ranked "in order" according to some measurable characteristic. The system of grades in the Army is an ordinal system. One can say Sergeant>Corporal>Private (where the symbol ">" means "greater than"). If the Private has one stripe, the Corporal has two and the Sergeant has three, the important thing is the relative number. That is, the same relationship would hold if the Private had one stripe, the Corporal four, and the Sergeant ten. Appropriate statistics now include the median, since it is defined as that point along the scale where there is an equal number of higher and lower ranking measurements. Hypothesis testing may be accomplished by any of the non-parametric methods based on ranking plus those allowed for nominal data. Again, the mean and standard deviation cannot be used. For example, one rank severity of malfunctioning equipment, particularly if you are speaking of its effect on a system or performance of a job. But how would you get the average? The problem is defining the scale between levels of severity,

they all seem to be different. It's like trying to define average pain.

Interval. This next higher level is everything that the ordinal scale is, plus there is a consistent unit of measurement defined between the values. Our temperature scales are of this type. The units of measurement may be different, and may be measured from different arbitrarily chosen zero points, but differences in values (intervals) on the scale have meaning. Also, the difference between $10^{\circ}F$ and $20^{\circ}F$ is the same as the difference between $110^{\circ}F$ and $120^{\circ}F$. The same is true for Centigrade. All the previously mentioned statistics may be used plus all those based on the mean and standard deviation. An average temperature makes sense, now.

Ratio. This is the highest level of measurement. A shortcoming of the interval scale is the absence of a natural zero point. Due to this lack, one cannot say that $20^{\circ}F$ is twice as warm as $10^{\circ}F$. On a ratio scale, however, one may say that two inches is twice as long as one inch. The unit of measurement again may be picked arbitrarily, but now a ratio in one system of units is the same as a ratio in any other system, thus the name ratio scale. For instance, a length of rope may be twice as long as another no matter what unit is used. All previously mentioned statistics are appropriate here including the geometric mean and coefficient of variation, which will not be needed in this guide.

Hypothesis Testing

The bulk of what statisticians do can be regarded as testing hypotheses. Webster gives the definition of hypothesis as the following:

> ....an unproved supposition or proposition (working hypothesis) tentatively accepted to explain certain facts or to provide a basis for further investigation.

The user does not know what the true "state-of-nature" is, so he makes a supposition, then checks (tests) whether the data available supports the hypothesis. This hypothesis is called the null hypothesis, $H_o$, and he either accepts or rejects this null hypothesis. When he rejects $H_o$, he rejects it in favor of some alternative hypothesis $H_1$. Then he takes some action on the basis of whether he decided to accept or reject $H_o$. We now have two states-of-nature and two actions possible. If these are arranged as shown in Fig 1, it is obvious that there are four outcomes possible, two correct decisions and two incorrect

| | STATES OF NATURE | |
|---|---|---|
| Action | $S_o$: $H_o$ True | $S_1$:$H_o$ Not True |
| Accept $H_o$ | Correct Decision | Incorrect Decision Type II Error |
| Reject $H_o$ | Incorrect Decision Type I Error | Correct Decision |

Fig 1. Type I, Type II Errors

ones (errors). The error of rejecting a true null hypothesis is called a Type I error, and its probability of occurrence is called $\alpha$. Likewise the error of accepting a false null hypothesis is called a Type II error and its probability of occurrence is called $\beta$.

Of course, there is more to the "action" than just accepting or rejecting $H_o$. For instance, let us say our states-of-nature are whether it will be fair (under $H_o$) or rain (under $H_1$) and the actions whether we will leave our umbrella ($A_o$) or take it ($A_1$). Our null and alternative hypotheses are:

$H_o$: It will be fair.

$H_1$: It will rain.

Our diagram will then look like Fig 2.

The decision here would be made on the basis of a weather report. In statistics we don't have to rely on a prediction, usually, but instead on a sample of the real but undescribed population, which fortunately can at least

| Action | STATES OF NATURE $S_o$: $H_o$ True | $S_1$: $H_o$ Not True |
|---|---|---|
| Accept $H_o$ | Correct Decision | Get Wet<br>$\beta$=p{Getting Wet} |
| Reject $H_o$ | Look Silly<br>$\alpha$=p{Looking Silly} | Correct Decision |

Fig 2. Umbrella Example

be quantified. Actually, when we say "hypothesis" in statistics, we mean a frequency distribution (Ref 10:231). When we accept or reject $H_o$, then, we are accepting or rejecting whether our sample came from the hypothesized population distribution. If our sample mean $\bar{y}$ is too much different from a hypothesized population mean $\mu$, we must reject the hypothesis. Assuming normality the tabulated value of the test statistic $z_\alpha$ is a measure of how close $\bar{y}$ and $\mu$ should be at a given $\alpha$. The computed value of the z-statistic

$$z = \frac{\bar{y}-\mu}{\sigma\sqrt{n}} \tag{2-1}$$

(from our data) is then compared to the <u>tabulated</u> value and the hypothesis is accepted or rejected accordingly.

A word here about measurement scales. To use Eq (2-1), the data must be at least interval (i.e., interval or ratio), since the mean $\mu$ and standard deviation $\sigma$ are involved. Testing of hypothesis about binomial populations can usually be reduced to testing some hypothesis about the parameter p, the probability of occurrence (Ref 2:101). That is, one assumes a distribution with parameter p, then checks whether the sample's frequency of occurrence is consistent with the p of the distribution. These binomial tests may be done with ordinal or even nominal data.

Since we are dealing with distributions, things are more complicated than our umbrella example (Fig 2) shows, but we may use it to illustrate another very important point.

We have said nothing about which type error we most wish to avoid. If we embarass easily, we may wish to avoid looking silly carrying an umbrella, so we concentrate on keeping that error as small as possible. But what if we were going directly to a job interview? If we got wet we would make a bad impression. Normally what is done here is change our $H_o$ and states-of-nature as shown in Fig 3, while still keeping our level of significance (probability of Type I error), $\alpha$, at its same small value. In this context $\alpha$ is also called the critical level, where we reject $H_o$.

| Action | STATES OF NATURE<br>$S_o$: $H_o$ True<br>(It Will Rain) | <br>$S_1$: $H_o$ Not True<br>(It Will Be Fair) |
|---|---|---|
| Accept $H_o \Rightarrow$<br>$A_o$: Take Umbrella | Correct Decision | Look Silly<br>p{Looking Silly}=$\beta$ |
| Reject $H_o \Rightarrow$<br>$A_1$: Leave Umbrella | Get Wet<br>p{Getting Wet}=$\alpha$ | Correct Decision |

Fig 3. $H_o$, $H_1$ Switched

Manipulating $H_o$ to reduce the error we wish most to avoid is analogous to burden-of-proof considerations in criminal justice. A person is innocent until proven guilty "beyond a reasonable doubt". We wish to avoid punishing an innocent man so much we are willing to risk letting a guilty man go free. The Type I error is the one we control

to a certain specified level so we set up our hypotheses so that the burden-of-proof is on the prosecution. Likewise, if we embarass easily, we want to leave the umbrella at home unless we are convinced (beyond a "reasonable doubt") that it will rain. We could have just as well set $H_o$ up so that we would take the umbrella unless we were convinced (beyond a "reasonable doubt") that it will be fair.

Again, with statistics we are somewhat better off than the job-hunter and the weather. With our sample mean $\bar{y}$, population standard deviation $\sigma$, and our formula, Eq (2-1) we can get both a decision rule (for action) and compute the probability $\beta$ of the Type II error. Remember, we have already specified what we want $\alpha$ to be. A hypothetical example will illustrate this.

Suppose a commander had a campaign to improve emergency procedure scores in his wing. He wishes to throw a party if the overall average came up to their goal of 90% Naturally, as mentioned before, we are dealing with a distribution, and the 90% is the mean of a distribution. The commander is hypothesizing that $H_o$: $\mu$=90% and wishes to test this hypothesis. Liking parties, he wants the risk of rejecting $H_o$ (when it is actually true) and not throwing the party to be small, say $\alpha$=.05. To test the hypothesis he takes a sample of 9 crew members (n=9) and gives them the test.

| | STATES OF NATURE | |
|---|---|---|
| Action | $H_o$ True: Distribution is $S_o$ | $H_o$ Not True: Distribution is $S_1$ |
| $A_o$ (Party) | No Error | $\beta=P(A_o\|S_1)$ |
| $A_1$ (No Party) | $\alpha=P(A_1\|S_o)$ | No Error |

Fig 4a. Party Example

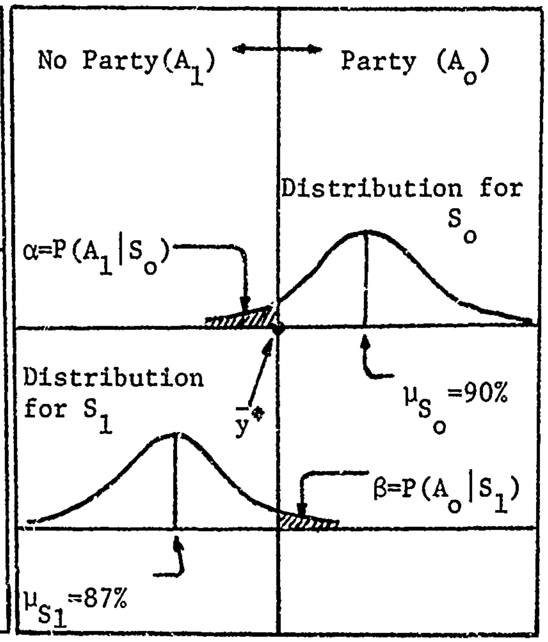



Fig 4b. Distributions

Our diagram is as shown in Fig 4a. It can be depicted using distributions as shown in Fig 4b. Normally in statistics our alternative hypothesis in this situation would be $H_1:\mu<90\%$ since our action after rejecting $H_o$ and accepting $H_1$ would be the same at several values of $\mu<90\%$ (action $A_1$ or no party). However, all lower values of $\mu<90\%$ affect only $\beta$, and have no effect on our chosen $\alpha$. The commander is only interested in whether he should have the party or not, not how bad his crewmembers' averages really were. If he were, he might well have used a different null hypothesis.

The alternative hypothesis $H_1$: $\mu=87\%$ above, was picked strictly for illustration. This specific value will enable us to compute the probability of a Type II error $\beta$. The straight vertical-line notation $P(A_1|S_o)$ means the probability of <u>not</u> throwing the party ($A_1$) <u>given</u> that the <u>true</u>

populati mean is 90% ($H_o$ true). This is our probability of a T e I error α. Here α=.0 equals the probability P(A $S_o$). This is called the conditional probability of $A_1$ ver the state-of-nature $S_o$ exists.

The comm wants a value $\bar{y}^*$ somewhere between 87 and 90 such tha hen a sample $\bar{y}$ is taken from his wing and it is better than $\bar{y}^*$, ($\bar{y}>\bar{y}^*$) he will give the party and if $\bar{y}<\bar{y}^*$ he will not, i.e., a decision rule. If he knows from experience that σ=3% and he plans to test a sample of size n=9, he can compute this critical value $\bar{y}^*$ using formula, Eq (2-1) as follows:

$$z_{\alpha=.05} = \frac{\bar{y}^*-\mu_{S_o}}{\sigma/\sqrt{n}}$$

$$1.645 = \frac{\bar{y}^*-90}{3/\sqrt{9}}$$

Solving for $\bar{y}^*$, $\bar{y}^* = \underline{88.4}$

This says if his sample mean $\bar{y}\geq 88.4\%$ he accepts $H_o$ and gives the party, but if $\bar{y}<88.4\%$ he rejects $H_o$ and does not. Put another way, there is only an α=.05 probability that the sample came from a population with a mean of 90% if the sample mean is less than 88.4%.

We can also use formula, Eq (2-1) to compute our β associated with this particular alternative hypothesis or state-of-nature. We compute z:

$$z_\beta = \frac{\bar{y}-\mu_{S_1}}{\sigma/\sqrt{n}} = \frac{88.4-87}{3/\sqrt{9}} = 1.4$$

The area under this part of the lower curve in Fig 4b associated with the computed value of Z is .081. So we have $\beta$=.081. We can plot various values of $\beta$ for various alternative hypotheses $H_1$. They will look something like the lines in Fig 5. When the curve (called an Operating Characteristic or OC curve) is high near the value $H_o$ and low away from $H_o$ it indicates the ability of the decision rule to distinguish between the hypothesized state-of-nature $S_o$ and possible alternative states-of-nature $S_1$ associated with the many alternative hypotheses. The value that determines the steepness of the curve (quality or power of decision rule) is the sample size n, as indicated in Fig 5. Operating characteristic curves are given in

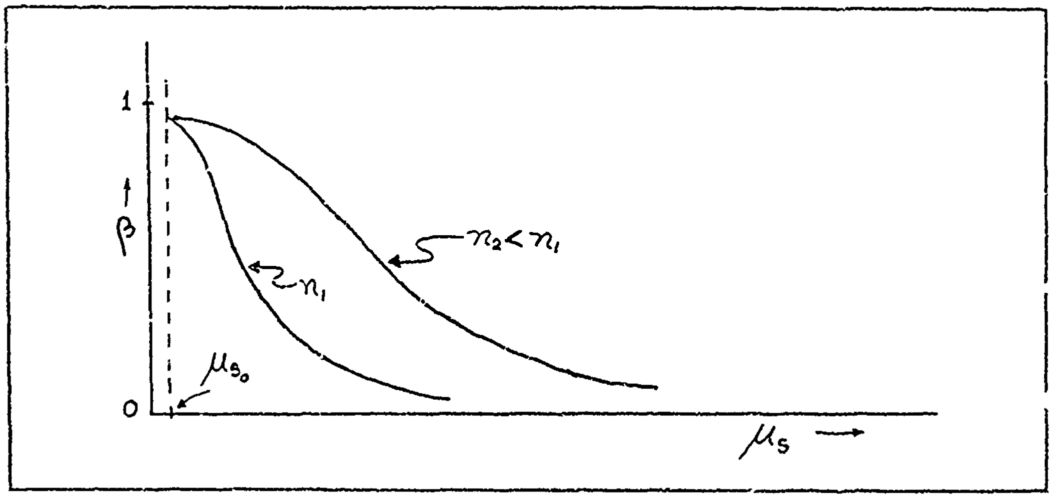



Fig 5. Operating Characteristic Curves

most statistics books. Normally one is not as concerned about the value of $\beta$ as he is the value of $\alpha$, but he should realize there is a tradeoff between sample size and the risk of a Type II error for fixed $\alpha$. For a fixed n there is a trade off between $\alpha$ and $\beta$.

# III. Methods from the Literature

## Purpose and Overview of Chapter

Purpose. The purpose of this section is to present and discuss the methods the author feels are applicable to measurements handled by Air Force trend analysis and Standardization/Evaluation offices. These methods will be presented in much the same manner as in representative textbooks. If the reader/user wishes to skip this chapter and go directly to the "guide" due to lack of time, interest, or thinks he may not have sufficient background to follow, he may do so here. Certain important assumptions (sample size, types of measurements, etc.) will be repeated there, to re-emphasize and to assure all are considered in gathering and analyzing the data. It is highly recommended, however, that this section be reviewed sometime, because it may give the user a better "feel" for the methods than just following the steps of the algorithm.

Time-Series. Data classified by time is called Time-Series data (Ref 7:18). This research effort does not strictly address only time-series methods, but these will be the methods concentrated on. Although the word "trend" makes one think of changes over time, those put in the job of trend analysis find themselves responsible for other types of statistical manipulations, such as correlation of two independent variables. It is possible attempts are

made in the field to describe correlation of more than two independent variables, but these methods will not be covered.

Overview. Due to an overwhelming number of questions from the field about statistical significance of trends, a description of the concept of control charts will be given. It is planned to use the control chart with all methods in the guide except correlation and non-parametric methods. Straight line least-squares-best-fit (LSBF), the most visually appealing method, will then be covered. The LSBF method will be extended to a non-linear model, the fitting of a parabola. Correlation between two variables will also be covered, with emphasis on the pitfalls of correlation vs. causality.

The next processes covered will be those used primarily in the business world, moving averages and exponential smoothing. These methods are widely used in business forecasting and provide a simpler, quicker method of reacting to, or computing, new data points in the model.

Lastly, non-parametric methods will be presented. These are the most general and sometimes the easiest to apply, but as a result are not always as powerful. Cox and Stuart, Kendall's and Spearman's tests for trend will be addressed.

## Control Charts

Air Force Standardization/Evaluation requires crew

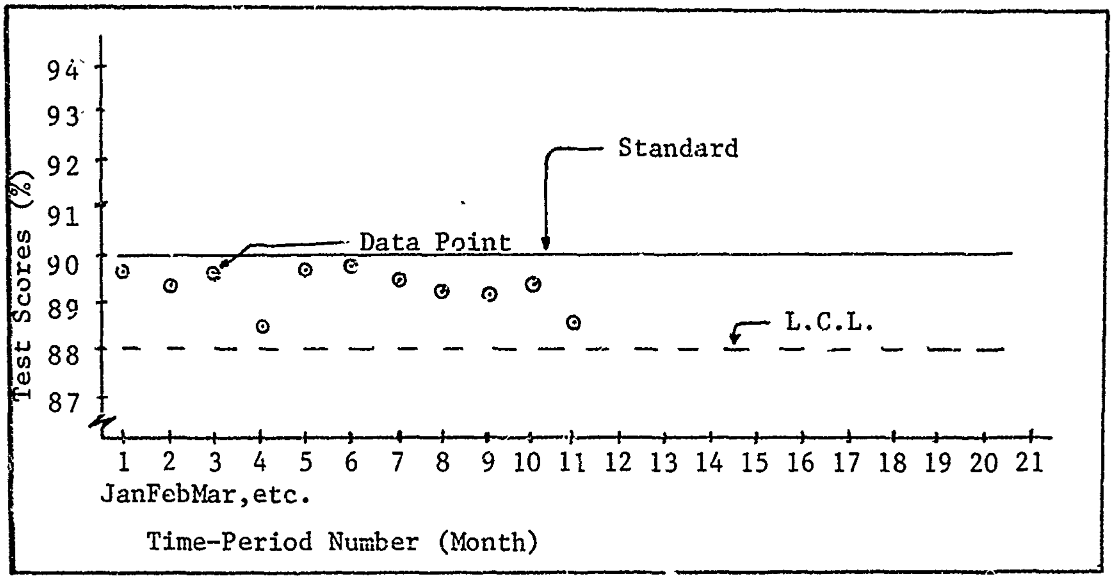



Fig 6. Control Chart

members to perform at some given standard. They are tested periodically against that standard. This process is quite analogous to manufacturing, where a quality-control inspector samples machined parts to determine if, on the average, they are within tolerances.

Let us say we wish to maintain a standard of 90% correct responses on Emergency Procedures examinations. We could construct a chart like the one shown in Fig 6. The discontinuity near the origin is strictly to save space, and not to exaggerate small vertical differences. In industry, several kinds of control charts are used. Quality control engineers look for trouble when a sample point falls beyond the control limits. One source mentioned that even when the points fall between the control limits, a trend or some other systematic pattern may serve notice that action should be taken to <u>avoid</u> serious trouble (Ref 4:421).

Normally control charts have both an upper control limit (UCL) and a lower control limit (LCL). In the case of test scores, however, commanders would most likely be interested in scores displaying a trend toward the LCL, unless he was looking for some policy's positive effect on already good scores.

Let us say he is interested in just the LCL. If there were a known population mean and standard deviation, $\mu$ and $\sigma$, we could establish $LCL=\mu-z_{\alpha}\frac{\sigma}{\sqrt{n}}$. The value z may be thought of as the number of sample standard deviations from the mean of the sampling distribution. The parameter $\alpha$ is the probability a sample mean would lie at least z standard deviations from the presumed or hypothesized population mean, or below the LCL. The "n" stands for the sample size. Here we are assuming that $\mu$=90% and we wish to know when scores begin to deviate significantly from this standard or assumed population mean.

We very rarely know the population standard deviation for sure, but if our sample size is large enough, $n>30$, we may substitute the sample standard deviation "s" for $\sigma$ (Ref 5:169). Also, if n is large, the distribution need not be normally distributed.

If n is <u>not</u> large, but the assumption of normality is a reasonable one, we may use the expression $LCL=90-t_{\alpha}\frac{s}{\sqrt{n}}$ where t is the value of a random variable having the Student-t distribution with the parameter $\nu=n-1$. The t-distribution is well tabulated and widely available.

The parameter "$\nu$" is called Degrees of Freedom (DF). The parameter $\alpha$ means the same as in previous treatments.

In order to have straight horizontal lines for control limits, the quantity standard deviation s must be assumed to be nearly constant. If this is not a reasonable assumption, there will have to be limits computed and plotted for each time period. This does not invalidate the use of the control chart, however, but it does make it a bit more difficult and cluttered.

We now have a standard with which to compare our trend and a value (LCL) where our measurements or observations (test scores, say) start to become significant at some level. The parameter $\alpha$, in this context, again, is called the level of significance.

Linear Regression or Least-Squares-Best-Fit

Many times a series of data points suggests a straight line. In fact, the eye is known to attempt to see "patterns" that may not even be there, so a person may easily see a line in a string of points over time. In mathematical terms, this says the relationship between x (the independent variable) and y (the dependent variable) or the regression of y on x, can be described by the relation $y=\beta_0+\beta_1 x$ where a is the y-axis intercept and b is the slope of the straight line. In statistical terms, we say there is such a line that best fits the data. Here y is a point on the line, and though it may not be one of our

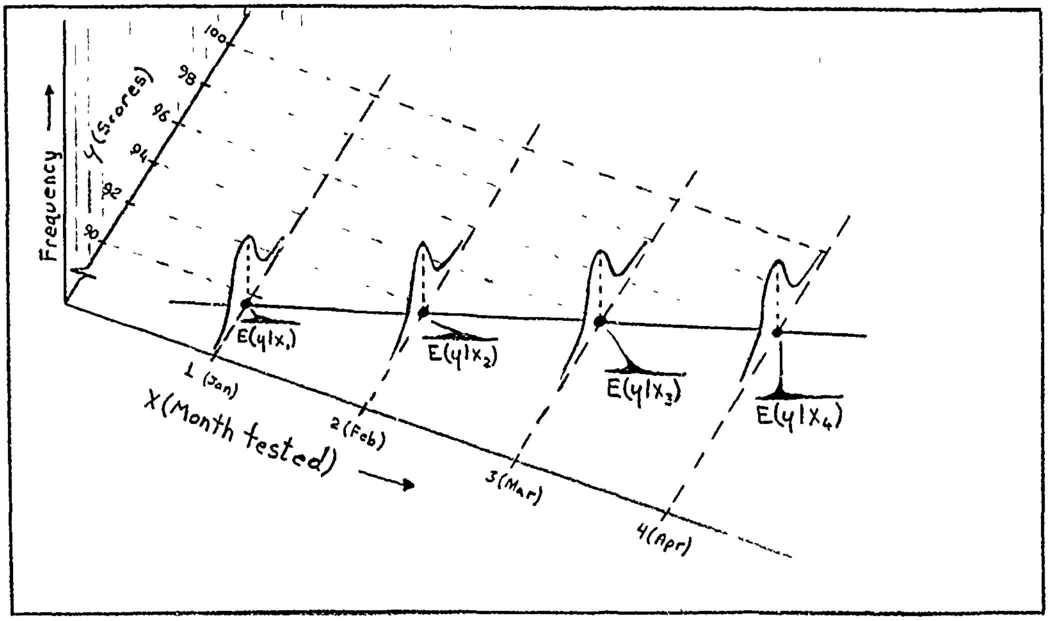



Fig 7. Regression Line with No Error

observations, it is (or assumed to be) the mean of the distribution of y's. Actually, it is the mean of the y's given a value of x, or the Expected Value $E(y|x)$. See Fig 7.

In general, an individual observation y, since it is a measurement, may contain some error; furthermore, the true but unknown relationship may not be exactly linear, so there is one more term, $\varepsilon$, which contains all the errors. That is, $\varepsilon$ contains both random errors of measurement and systematic errors due to an incorrect relationship being fitted.

The relation can now be described by the equation

$$y_i = E(y|x_i)$$

$$\text{or} \quad y_i = \beta_o+\beta_1 x_i+\varepsilon_i \qquad (3\text{-}1)$$

as mentioned above. We can always theoretically choose $\beta_o$ such that the mean of the random variable $\varepsilon$ is zero with certain assumptions mentioned later. The method of least squares estimates the parameters $\beta_o$ and $\beta_1$ by minimizing the vertical distance between the line and our points. If e, a, and b are the estimators of our $\varepsilon$, $\beta_o$, and $\beta_1$, then we are finding the values that minimize this equation:

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(y_i - (a+bx_i)\right)^2 \tag{3-2}$$

From here on, $\sum_{i=1}^{n}$ or just $\Sigma$ will mean summation over the n subscripted arguments.

Taking the partial derivatives, setting them equal to zero, and solving them simultaneously, give us the estimators:

$$a = \bar{y} - b\bar{x} \quad \text{and} \quad b = \frac{S_{xy}}{S_{xx}} \tag{3-3}$$

where $\bar{x}$ and $\bar{y}$ are the means of x and y and:

$$S_{xx} = n\Sigma x_i^2 - (\Sigma x_i)^2 \tag{3-4}$$

$$S_{xy} = n\Sigma x_i \cdot y_i - (\Sigma x_i)(\Sigma y_i) \tag{3-5}$$

The Gauss-Markov Theorem states that among all unbiased estimators for $\beta_o$ and $\beta_1$ which are linear in the $y_i$, the

least square estimators a and b have the smallest variance and are the most reliable in the sense that they are subject to the smallest chance variation (Ref 10:396). This theorem assumes the following properties:

1. The $x_i$'s are fixed (months, for instance).
2. The $y_i$'s are random variables (distribution not specified).
3. The $\varepsilon$ are independent (not necessarily normal) but $Var(\varepsilon)=Var(y)$.

These should be reasonable assumptions for trending of such things as test scores over time.

One even more basic assumption, however, is that the time periods are independent of each other. Normally one can make this assumption if there are no cycles or seasonal-type variations over time. If there are such cycles over time, other time-series methods may be used to minimize their effect. This is not the only reason why one may wish to use other methods. Computing parameters for a new regression line each time period gets tedious. The section following correlation deals with such simpler methods, starting with moving averages. In addition, the assumptions underlying these methods are not so restrictive.

The case where the regression of y on x is linear leads to fitting other non-linear curves to data. For example, some functions can be transformed into linear functions such as $y=a \cdot b^x$, which can be transformed into

$\log y = \log a + x \log b$ by taking the logarithm of both sides of the equation. Likewise, $y=a \cdot x^b$ can be transformed into $\log y = \log a + b \log x$, again by taking logarithms.

We will not be dealing with these transformations in the guide, but will be using a polynomial regression, that is, the fitting of a parabola of the type $y=a+b \cdot x+c \cdot x^2$. The method is the same as for the straight-line model, except the following three equations must be solved for the three unknowns a, b, and c:

$$na+(\Sigma x)b+(\Sigma x^2)c = \Sigma y \tag{3-6}$$

$$(\Sigma x)a+(\Sigma x^2)b+(\Sigma x^3)c= \Sigma xy \tag{3-7}$$

$$(\Sigma x^2)a+(\Sigma x^3)b+(\Sigma x^4)c= \Sigma x^2y \tag{3-8}$$

Correlation. In correlation analysis both variables are random variables with some distribution. Relating this to Fig 7 from the linear regression section, we say not only is there a distribution of y's given a value of x, but there is also a distribution of x's given a value of y. The distribution at any one point might look something like Fig 8. Such distributions are referred to as joint or bivariate distributions. The regression analysis of y on x (previous section) implied that y was dependent on x. Correlation analysis treats the data both ways, symmetrically, and is neutral concerning the direction of the dependency. For our uses, this means it is up to the

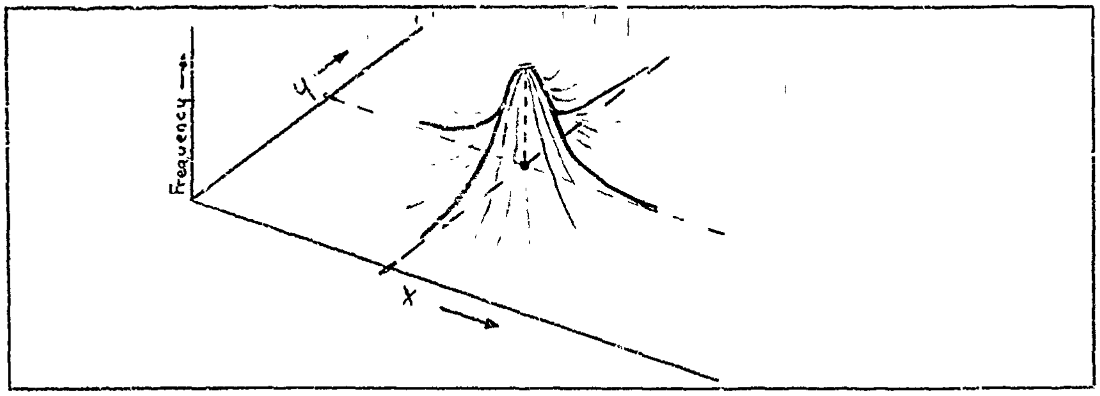



Fig 8. A Joint Distribution

decision-maker to speculate on the cause-effect relationship, as the numbers just give the magnitude of the co-relationship.

For such distributions the correlation coefficient $\rho$ is defined by the equation

$$\rho^2 = 1 - \frac{\sigma^2}{\sigma_2^2} \qquad (3\text{-}9)$$

where $\sigma^2$ is the variance of the conditional distribution $g_2(y|x)$ and $\sigma_2^2$ is the variance of the marginal distribution $f_2(y)$. What is meant by the conditional and marginal distributions is shown in Fig 9. The formula is derived in much the same manner the parameters were for simple linear regression, except that errors are minimized from both directions. Note that $\rho^2$ can be written:

$$\rho^2 = 1 - \frac{\sigma^2}{\sigma_2^2} = \frac{\sigma_2^2 - \sigma^2}{\sigma_2^2} \qquad (3\text{-}10)$$

We know $\sigma^2$ is a measure of the variation of the y's when x is held fixed while $\sigma_2^2$ is a measure of the variation of

Conditional Distribution: $g_2(y|x=49)$

Height, X Inches

Marginal Distribution: $f_2(y)$

| Weight, Y Pounds | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | $f_Y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 40 | 1 | | | | | | | | | | 1 |
| 41 | | 1 | 1 | 1 | | | | | | | 3 |
| 42 | | 2 | 2 | 2 | 1 | | | | | | 7 |
| 43 | | | 3 | 3 | 2 | | | | | | 8 |
| 44 | | | 2 | 4 | 2 | 1 | | | | | 9 |
| 45 | | | 1 | 3 | 5 | 2 | 1 | | | | 12 |
| 46 | | | | 3 | 4 | 3 | 2 | | | | 12 |
| 47 | | | | 2 | 4 | 2 | 1 | 1 | | | 10 |
| 48 | | | | | 2 | 1 | 2 | 1 | 1 | | 7 |
| 49 | | | | | | | 1 | 1 | 1 | 1 | 4 |
| $f_X$ | 1 | 3 | 9 | 18 | 20 | 9 | 7 | 3 | 2 | 1 | 73 |

Fig 9. A Joint Distribution Frequency Table

the y's when x is <u>not</u> held fixed. Our $\rho^2$, then, is a measure of the <u>proportion</u> of the variation in the y's that can be attributed to a linear relationship with x (Ref 1:323).

The correlation coefficient $\rho$ may be estimated by the sample correlation coefficient

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} \tag{3-11}$$

where $S_{xx}$, $S_{xy}$, and $S_{yy}$ are defined as in the last section. The null hypothesis $H_o$: $\rho=0$ may be tested using the z-statistic

$$z = \frac{\sqrt{n-3}}{2} \cdot \ln \frac{1+r}{1-r} \qquad (3\text{-}12)$$

Several cautions are appropriate here. In addition to the same assumptions about normality and independence (of errors) required for straight-line LSBF, one must be aware that correlation coefficients based on relatively small samples are generally not very reliable (Ref 4:326). Also, r is a measure of the strength of a linear association between the random variables. As illustrated in Fig 10, r could show up as close to zero when in fact there is a strong (but non-linear) relationship (Ref 4:327). Thirdly, as mentioned in the beginning of this section, a significant correlation does not necessarily imply a causal relationship between the variables.

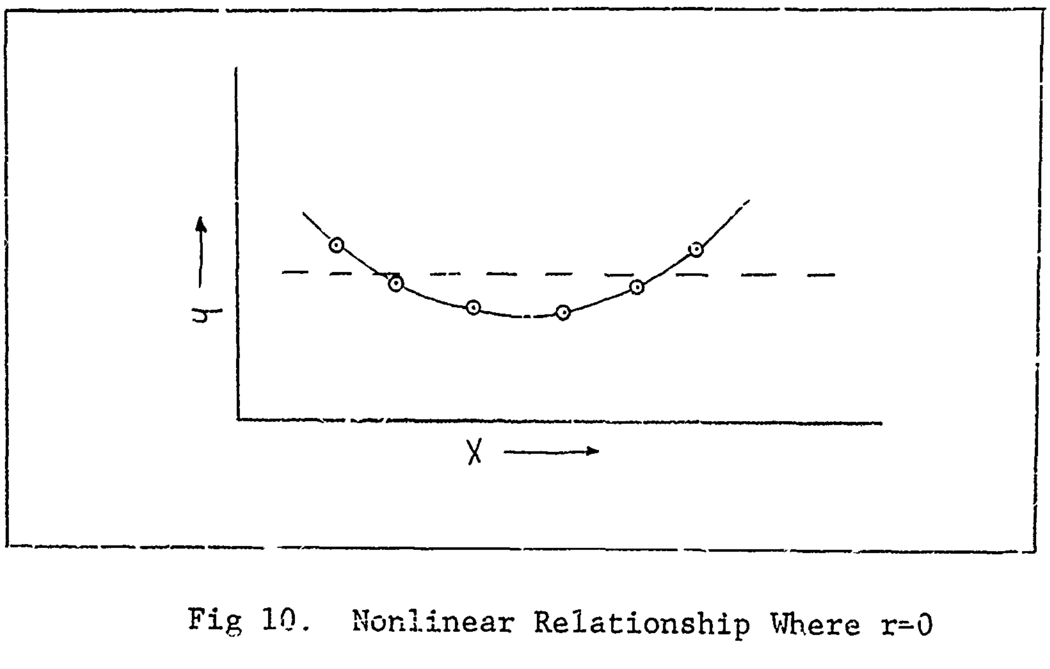



Fig 10. Nonlinear Relationship Where r=0

Types Used in Business Forecasting

The methods in this section will be particularly useful when there are periodic fluctuations over time. For instance, if low visibility weather affects visual or radar bombing, then it is conceivable that bombing scores may have cyclical or seasonal variations over several years. In our linear regression, we assumed that time (independent variable) was also independent of itself. Another way of putting this is "no autocorrelation". In our example above, the time of the year has traits that are definitely not independent of whether the last few months were the end of Winter or the end of Summer. Our LSBF line could indicate an erroneous "trend" if it started, say, in the Spring and went for several months.

The following methods tend to "dampen out" such cyclical effects. In addition, fewer calculations are necessary when a new data point is obtained, and some methods permit a forecaster to place more weight on current data, as opposed to equal weighting of all data.

Although no specific assumptions are necessary for applying these methods, when they are used with control charts to test significance of trends, all control chart assumptions must be satisfied.

Moving Averages. The method of simple moving averages simply takes the last N measurements, computes their arithmetic mean called $m_i$ and this arithmetic mean becomes the

forecast for the next T periods. As each new data point is acquired, it is figured in while the oldest one is eliminated. A small number N (say 3 to 5) will respond more quickly to large, permanent changes in the response variable than larger N's (more than 10), but will also respond <u>too</u> quickly to anomalies or noise (Ref 8:88). The moving average formula for time period $x_i$ is:

$$M_i = \frac{\sum_{i=1}^{N} y_i}{N} \tag{3-13}$$

$$\text{or} \quad M_i = M_{i-1} + \frac{y_i - y_{i-N}}{N} \quad \text{(short formula)} \tag{3-14}$$

where the $y_i$'s are the observed responses at time period $x_i$. The short formula may be used after computing the first $M_i$.

Double moving averages improve the response to changes in trend while keeping the risk of responding to an outlier small. Here, two moving averages are computed. The moving average $M_i$ is computed as in Eqs (3-13) or (3-14), but is now called $M_i^{[1]}$, and $M_i^{[2]}$ is computed as the average of the last N $M_i^{[1]}$'s. These two averages are then used to determine an equation for values of the dependent variable a certain number of periods T=1,2,... in the future. This equation is

$$y_{i+T} = a_i + b_i \cdot T \tag{3-15}$$

where $$a_i = 2M_i^{[1]} - M_i^{[2]}$$

and $$b_i = \left(\frac{2}{N-1}\right)\left(M_i^{[1]} - M_i^{[2]}\right)$$

This double moving average is quite good for linear trends but will not give accurate results when there is curvature in the data. While there is a triple moving average that addresses curvilinear data, triple exponential smoothing, covered in the next section, is normally used in that situation (Ref 8:90).

Exponential Smoothing. Moving average mehtods, when one is using a relatively large N (>200), can require storage of a prohibitive number of data points. Normally this is not a problem with the types of trending involved here, but this method is, nevertheless, more flexible in how it can react to new data. The following is a development of the general exponential smoothing model (Ref 8:91). In the short formula if i were 6, for instance, and N were 5, we would have

$$M_6^{[1]} = M_5^{[1]} + \frac{y_6 - y_1}{5} \tag{3-16}$$

If we stored none of the data, a value of $y_1$ would not be available. Our best estimate of $y_1$ would then be $M_5^{[1]}$. By using this in our equation, we get

$$M_6^{[1]} = M_5^{[1]} + \frac{y_6 - M_5^{[1]}}{5}$$

$$M_6^{[1]} = M_5^{[1]} - \frac{1}{5} M_5^{[1]} + \frac{y_6}{5}$$

$$M_6^{[1]} = \frac{4}{5} M_5^{[1]} + \frac{y_6}{5} \qquad (3\text{-}17)$$

For the general case,

$$M_i^{[1]} = \frac{1}{N} y_i + \left(1 - \frac{1}{N}\right) M_{i-1}^{[1]} \qquad (3\text{-}18)$$

The above equation could be used to calculate $M_i$ if no data were stored. If we let $r = \frac{1}{N}$ and $S_i^{[1]} = M_i^{[1]}$ and substitute these into the last equation, we arrive at the basic exponential model:

$$S_i^{[1]} = r\, y_i + (1-r) S_{i-1}^{[1]}$$

or $$\begin{pmatrix}\text{New}\\ \text{Estimate}\end{pmatrix} = r\begin{pmatrix}\text{new}\\ \text{data}\end{pmatrix} + \left(1-r\right)\begin{pmatrix}\text{previous}\\ \text{estimate}\end{pmatrix} \qquad (3\text{-}19)$$

The term r, is called the smoothing constant. In general, r should lie between 0.01 and 0.30, but the forecaster should not hesitate to use a value outside this range if it gives better results. It could be <u>very</u> profitable to periodically pick an arbitrary point in the past, apply forecasting techniques and see how well they did. This can

also be a tool for picking or changing your own value of r. The most important fact concerning r is that a larger r places more importance on the most recent data (Ref 8:92).

An argument similar to the one for going from the simple to the double moving averages model, Eq (3-13) to Eq (3-15) will be used to go from simple to double exponential smoothing. The basic exponential smoothing model is best when there is very little trend. To describe or react to a linear trend in data, a straight-line model similar to Eq (3-15) should be used. Such is the case with the parameters $a_i$ and $b_i$ developed from the double exponentially smoothed statistic $S_i^{[2]} = rS_i^{[1]}+(1-r)S_{i-1}^{[2]}$. That is, the following equation:

$$y_{i+T} = a_i + b_i \cdot T$$

where $$a_i = 2S_i^{[1]} - S_i^{[2]}$$

and $$b_i = \frac{r}{1-r}\left(S_i^{[1]} - S_i^{[2]}\right) \tag{3-20}$$

Again, if the observations suggest a curvature of some sort, one would assume a quadratic model may be appropriate. Again, a new statistic is needed, the triple exponentially smoothed statistic $S_i$, where

$$S_i^{[3]} = rS_i^{[2]} + (1-r)S_{i-1}^{[3]} \tag{3-21}$$

The model itself is of the form:

$$y_{i+T} = a_i + b_iT + c_iT^2$$

where

$$a_i = 3S_i^{[1]} - 3S_i^{[2]} + S_i^{[3]}$$

$$b_i = \frac{r}{2(1-r)^2}\left[(6-5r)S_i^{[1]} - 2(5-4r)S_i^{[2]} + (4-3r)S_i^{[3]}\right]$$

$$c_i = \frac{r^2}{2(1-r)^2}\left[S_i^{[1]} - 2S_i^{[2]} + S_i^{[3]}\right] \qquad (3\text{-}22)$$

## Non-Parametric Methods

These methods are more generally applicable than those described in the first section, the parametric methods. If the data cannot be assumed to come from a normal distribution and sample sizes are small, these tests may be used. They may also be used if measurements are only nominal or ordinal. Since they are more general, it is to be expected that they will not be quite as powerful as the standard methods when both are applicable (Ref 2:170).

Binomial Tests. The first method presented is the Cox-Stuart test for trend. This test is based on a variation of the sign test, which in turn is based on the Binomial distribution. This chapter will not go into a discussion of the distribution, but assumes the reader understands that it would apply if we are dealing with only

two outcomes with probabilities p and 1-p. The outcomes may be success-failure, heads-tails, or, in our "sign" case, plus-minus. We will be testing whether the proportion of plus (or minus) signs is consistent with the binomial distribution possessing the assumed or hypothesized parameters. Binomial tests are so versatile and can be applied so simply, they are sometimes preferred over more powerful tests (Ref 1:95). This was an important consideration when choosing statistical tests for this guide.

Cox and Stuart Test for Trend. The Cox and Stuart test for trend (Ref 1:130) takes the sequence of random variables $y_1, y_2, \cdots, y_n$, arranged (in our case) in the order in which they were observed. Then these observations are grouped in pairs $(y_1, y_{1+c})$. That is, the observations are split in the middle and the first observation or measurement in the first half is paired with the first measurement in the second half, the second paired with the second, and so on. If there is an odd number of observations, the middle one is discarded. The number of pairs is called "n".

The number of pairs where the second entry is greater than the first could be replaced by a plus sign, those where the second entry is less, a minus. If the null hypothesis is that there is no trend, the number of plus signs should equal the number of minus signs. In terms of the binomial distribution, this means the probability

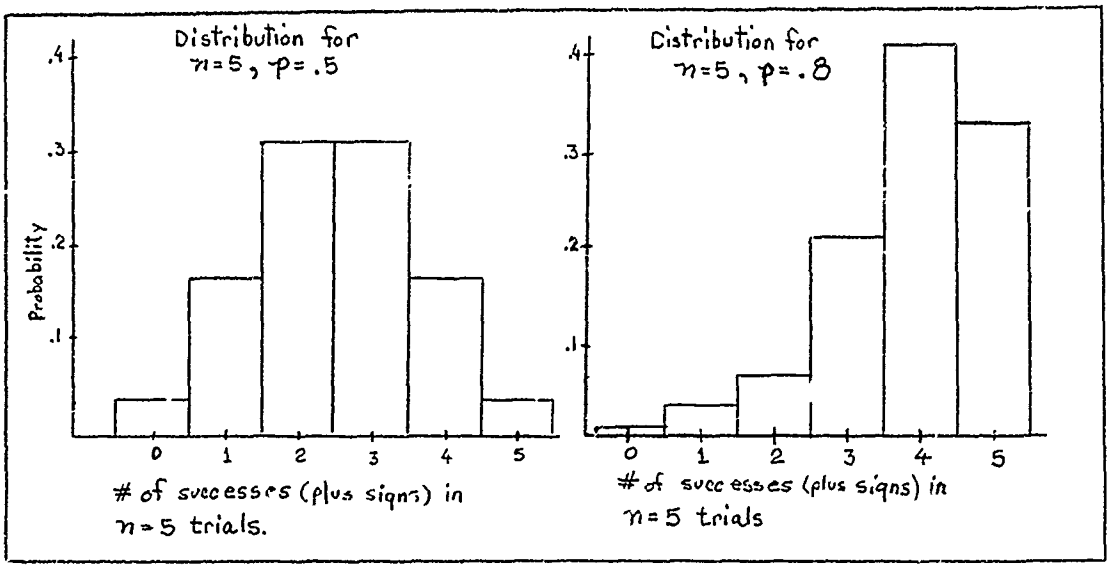



Fig 11. The Binomial Distribution

of a plus p(plus sign)=p(minus sign)=½. This hypothesis can be tested by asking whether such a value of T, the number of plus signs, lies in the critical region of the assumed binomial distribution, i.e., that distribution with p=½ and number of pairs "n". See Fig 11. The critical region is that point where T, the number of plus signs, is so great (or small), one must assume the sample of n pairs did <u>not</u> come from a population where p(plus sign)= p(minus sign)=½. That is, the hypothesis of no trend must be rejected. The size of the critical region, α, or the significance level, has basically the same interpretation as it did with the normal distribution in Chapter II.

When actually applying the Cox and Stuart test, it is sufficient to count the number of pairs and not convert to plus and minus signs as such. When counting the pairs where the second entry is greater than the first to get

the value T, one thing must be remembered. Ties, or equal entries, should not contribute to acceptance or rejection of $H_o$, so should not be counted.

In order to save space, tables usually stop at n=20, since the normal distribution is a fair approximation for n>20.

If there are cycles in the data and one knows which pairs of measurements appear at the same point in the cycle, then the overall trend can still be evaluated with the cyclical influence neutralized. For instance, if there were a yearly cycle, pairing the same months to compute the T-statistic would be done to test the presence of an underlying trend.

Efficiency, or power of the test for a given sample size, can be improved by discarding middle observations. For instance, the first ten observations could be paired with the last ten, or the first third paired with the last third. If loss of data is to be avoided, another variation is suggested (Ref 1:135). Pairing from the ends of the sequence $(y_1,y_n)$, $(y_2,y_{n-1})$, etc., also would exaggerate trend but still employ all data points. Usually loss of data <u>does</u> result in loss of power (Ref 1:133).

The Cox and Stuart test may also be used for correlation between two different measures. The paired measures are listed by the rank (in size) of either the first member of each pair or the second. If one member has less ties than the other, that member of each pair is preferred for

the ranking. The test for trend, then, is applied to the remaining member of the pair. An upward trend would indicate positive correlation, a downward trend would indicate negative correlation, and no trend would indicate zero correlation.

Kendall's Test for Trend. The next two methods, Kendall's and Spearman's tests for trend, use the concept of ranking. Ranking assumes that individuals (or airplanes, radar sets) can be arranged in order according to the magnitude of some measurable quantity. Customarily we denote the ranks by 1, 2, $\cdots$, n from smallest to largest.

Before defining Kendall's statistic, let us define concordant and discordant pairs. Two pairs are concordant if both members of one pair are larger than the respective member of the other pair. For example, the pairs (1,3) and (6,9) are concordant since both 6>1 and 9>3. Similarly, two pairs are discordant if one member is smaller and one is larger than the corresponding member of the other pair. For example, the pairs (1,6) and (9,3) are discordant.

If we let $N_c$ be the number of concordant pairs in a set of pairs, and $N_d$ the number of discordant pairs, Kendall's statistic $\tau$ is defined as the ratio:

$$\tau = \frac{N_c - N_d}{\frac{1}{2}n(n-1)} \tag{3-23}$$

where $\frac{1}{2}n(n-1)$ is the total number of pairs possible, often denoted by $\binom{n}{2} = \frac{1}{2}n(n-1)$ or n things taken 2 at a time.

A "$\tau$" of -1 means perfect reverse relationship, $\tau = 1$ means perfect positive relationship, and $\tau = 0$ means no relationship. Instead of computing $\tau$ itself, however, if Quantiles or critical values of $T = N_c - N_d$ are available, these may be used.

In practice, when this method is used for trending rather than correlation, counting concordant and discordant pairs is much easier than implied above. Since a time-sequence of measurements are in a ranking order already, such as time-period 1, 2, 3, $\cdots$, n, only the corresponding measurements (score for time-period 1, 2, etc.) need be considered. The steps to find T, $T = N_c - N_d$, are covered in detail in the guide. Significance testing is accomplished by comparing T to the critical value at significance level $\alpha$, as given in tables.

Spearman's Test for Trend. Spearman's test is quite similar to Kendall's, except that it uses the differences $R(x_i) - R(y_i)$ between the ranks, squares them, and takes their sum. Then the statistic is defined as follows:

$$\rho = 1 - \frac{6\Sigma\left(R(x_i) - R(y_i)\right)^2}{n^3 - n} \qquad (3\text{-}24)$$

The quantity $R(x_i)$, again, denotes the rank of the time-period number (1st period, 2nd period, $\cdots$, nth period) and $R(y_i)$ is the rank of the measurements. Instead of computing Eq (3-24), the sum of squares, S, is defined as:

$$S = \sum\left(R(x_i)-R(y_i)\right)^2 \tag{3-25}$$

and critical values of this test-statistic S are well tabulated and widely available. Significance-testing is accomplished by comparing the computed S with table values to determine if it lies in the critical region.

# IV. The User's Guide

## Introduction

This guide is written for those persons who are put in the job of Trend Analysis (TA) but have a limited statistics or mathematical background. It is written with an aircrew member in mind, one who is in a staff position, but is asked to do TA. Even those who have such background may not recognize certain problems as lending themselves to statistical solutions. Also, such persons may not have used the methods for so long they are no longer familiar (Ref 3:1). This guide should provide the user with a tool for easily determining the form and significance of trends. Then the user's expertise in his primary job can be applied more readily to the problem of explaining or reversing the trend.

Some may think of TA as strictly identifying weak areas in his realm of responsibility. This is fine, but the writer feels that if one is collecting evaluation scores (for instance) over time anyway, and has methods available for measuring trend, he may find doing TA on his raw data yields very valuable information. Others may have the data dumped in their lap, so to speak, and told to analyze it for significant trends. Or possibly some are told to set up a program of data collection that will permit identification of trends. This guide is written primarily for the one who has data dumped on him, but <u>all</u>

may find it useful. That a familiarization with the methods should give any user a feel for what collection and classification methods need be used.

To that first person the writer says he might also be surprised how often people apply "trend analysis" unconsciously. Whenever a briefing is given with comparisons made by graphs and charts the eye does some "pattern recognition" and the viewer tends to extrapolate or project those patterns into the future. So, why not quantify such things?

This guide is intended to be relatively painless. It is <u>not</u> assumed that the user has read the first chapters, but that he will at some time go back and skim over them at least. They cover some of the theoretical background used in the development of the guide. The writer asks forgiveness if it seems he is insulting anyone's intelligence by belaboring simple things. It is <u>not</u> his intention, he just wishes to cover everything in sufficient detail.

<u>A Word on Samples</u>

Basic to the methods here is the idea of a sample mean. It is nothing more than an average (computation is covered in Guidesheet # 1) and it is assumed everyone has an idea of what an average is. A sample mean is an average, but the word "sample" implies the presence of a larger "whole". The basic idea may be clear from an example. Let us say we take a sample of a group of radar-navigators'

bombing CEPs. If the sample average or mean is small (say 100 ft), this means the group is a part of a "skillful" whole, or population, a population that always has small CEPs. If the group's CEPs were large, there would be some question as to whether they are members of this hypothetical skillful population. This whole idea is called statistical inference, i.e., can we "infer" this whole population is skillful? Chapter II of this thesis covers this in more detail.

Usage of the term "population" in statistics is a carry-over from the days when statistics was applied mainly to sociological and economic phenomena. Nowadays it is applied to any set or collection of objects, actual or conceptual, and mainly to sets of numbers, measurements, or observations (Ref 4:160).

## Statistical Inference

Whenever we talk about inference, we need some measure of how confident we are about our statistical results. Statistical literature talks about confidence level, level of significance, confidence intervals, and not always with consistency. Chapter II of this guide talks about the driving concept behind all of these, the concept of the size of the critical region $\alpha$, or the probability of a Type I error. In short, it is nothing more than a measure of how confident one is of a relatively unsure thing. We are dealing with probabilities, not certainties, so we are

never sure of our answers. For this guide's purposes we will consider $\alpha$, the level of significance, as the probability of drawing a wrong conclusion from our data. Specifically, when we deal with hypotheses, it will be the probability of rejecting a true null hypothesis (see Chapter II). Normally $\alpha$ is chosen to be $\alpha=.05$, somewhat less often chosen to be $\alpha=.10$ or $\alpha=.01$. In these cases our confidence level would be the complement of $\alpha$ or $1-.05=.95$ and $1-.10=90$. These could be thought of as being 95% and 90% confident of our conclusions or answers. Put in gambling terms a confidence level (again the complement of significance level $\alpha$) of 95% would mean nineteen-to-one odds that you have made the right decision or that your sample value is close enough to the true value to use. Likewise, a confidence level of 90% ($\alpha=.10$) is odds of nine-to-one.

One may say here, "Why not be 99.9% or even 100% confident all the time?" The reason is you are dealing with a <u>sample</u> and can never be absolutely certain about the population unless your sample <u>equals</u> the population. This is sometimes possible, but usually outrageously expensive. This can be put another way by bringing up another type of error. This was done in the introductory Chapter II, page 8 , and it will be sufficient to say (I hope) that one increases the chances of this second type error as he chooses lower values of $\alpha$ for a given sample size.

## Before Picking a Method

Some things had to be assumed when this guide was compiled. First and foremost, implied in every Trend Analysis method (all methods except those dealing with correlation) is that some sort of consistent measurement is taken over time. It may be once a minute, once a flight, once every 2.73 hours or whatever. The assumption in this guide is that one is measuring the same thing each time-period.

For all methods it is assumed that a hand-held electronic calculator is available. The feature of floating-decimal is required to get any degree of accuracy. A simple four-function calculator of this type is barely sufficient and costs less than five dollars. The next desired features would be a square-root function and then a power function, either $x^2$ or $y^x$ or both. Memory feature would also be a handy asset.

Finally, it is assumed that one will plot his data over time, <u>then</u> choose the method for trending. The method of trend-description chosen should fit the pattern suggested by the time plot. See Guidesheet #2 for plotting instructions and example.

The following are notes and cautions that need to be noted <u>for each method</u>. Although many of these deal with the time-plot, they need to be mentioned here for emphasis. Furthermore, due to the final assumption above, one must

think of these in conjunction with the application of all the methods of trending. Notes:

1---At times there will be more past data than is necessary, or in your judgement should not be entered into the test (rules or method of measurement changed, for instance). If this is the case, re-number the data points so that time-period #1 ($x_i$=1) is the first data point you feel should enter into the computations. This was done in the example for Curvilinear Regression, Guidesheet #6, Step 3.

2---Do not discard points, unless they are mis-measured or otherwise invalid. If one is discarded for being invalid, or a measurement is missing for that time-period, re-number the table and time-plot leaving the discarded point out completely. See Guidesheet #2, Step 8.

3---As soon as a time-plot is made, connect plotted points with straight line-segments. This will give visual impact, making it easier to see general shape, cycles, etc., in the data. When they have served their purpose, however, erase them or they will interfere with the computed trend line.

4---Usually when a new measurement is made and recorded on table and time-plot, the trend-line must be recomputed. If the new point lies on or very close to the forecast line, however, this is not necessary. Also, the method used last may not be the best this time, so reevaluate each new time-period.

5---If the data suggests a curve-type trend, and there is no time to do extensive computations, a "french curve" may be accurate enough, and control chart evaluations are still valid.

Cautions

1---Do not project trend-lines too far into the future. They tend to become very inaccurate after 3 or so periods into the future, especially the trends described by curved lines.

2---Be careful to keep as many decimal places as possible in the computations, because subtraction accumulates round-off errors. Final answers may be rounded off.

3---Be extremely careful when using the Control-Chart method (Guidesheets #3 or #4) for evaluating trends in average test scores. If the test scores are percentages, they may be quite high, especially when one is talking about Stan/Eval test scores. Contrary to popular belief, such scores may not be normally distributed. This may be because no percentage scores can possibly be above 100% Such distribution may be highly skewed (unsymmetrical) even when the standard deviation is small, say 2. Then, with such a $\sigma$, a trend may be considered significant when scores deviate very little from the standard. In this case, methods 3 and 4 are invalid. Guidesheets 11, 12, and 13 would be valid, however.

4---No statistical test should be blindly followed. If a test given in this guide gives illogical or inconsistent results, try one of the non-parametric methods (11, 12 or 13). The problem is probably with the form of the assumed distribution, such as noted in caution 3 above.

5---Be careful of momentary deviations (a single measurement deviating very much from a standard. Normally one should never decide there is a trend based on one data point. Momentary deviations cannot be handled (explained) by statistical methods.

6---When testing for trends in a single measurement per time-period, use Guidesheets 11, 12, and 13. Control Charts may only be used for trends in averages.

GUIDESHEET #1. The Mean and Standard Deviation

Steps:

1---Count the total number of observations or measurements which are to be averaged. Call this total number "n".

2---Arrange measurements in vertical columns (as in example) with column headings as follows. Column 3 will be filled in later.

| Name or Observation Number $i$ | Measurement $y_i$ | Square $y_i^2$ |
|---|---|---|
| | | |
| | $\Sigma y_i=$____ | $\Sigma y_i^2=$____ |

3---Compute $\sum_{i=1}^{n} y_i = y_1+y_2+y_3+\cdots+y_n$, the sum of all the measurements. The space directly under column 2 may be used for $\Sigma y_i$.

4---Compute $\bar{y}$, the Mean of the n measurements, by dividing the sum $\Sigma y_i$ by the total number of observations n.

$$\text{Mean} \qquad \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

5---Square each measurement, $y_i^2=y_i \cdot y_i$ and record in column 3.

6---Compute the sum of squares of the observations $\Sigma y_i^2=y_1^2+y_2^2+y_3^2+\cdots+y_n^2$. The space directly below column 3 may be used to record the value of $\Sigma y_i^2$.

7---Compute the variance $s^2$ according to the following formula:

$$s^2 = \frac{n(\Sigma y_i^2) - (\Sigma y_i)^2}{n(n-1)}$$

8---Compute the Standard Deviation $s=\sqrt{s^2}$, that is, take the square root of $s^2$ in order to obtain s.

EXAMPLE for Mean and Standard Deviation (Guidesheet #1)

Steps:

(1)---Ten aircraft have flown this week against a certain target. It is desired to find what the average bomb score is. n=10.

(2)---

| Name or Observation Number | Measurement (bomb score in hundreds of ft) $y_i$ | Square $y_i^2$ |
|---|---|---|
| Acft # 1 | 0.5 | .25 |
| " # 2 | 21.3 | 453.69 |
| " # 3 | 6.9 | 47.61 |
| " # 4 | 11.0 | 121.00 |
| " # 5 | 5.7 | 32.49 |
| " # 6 | 7.2 | 51.84 |
| " # 7 | 10.5 | 110.25 |
| " # 8 | 11.8 | 139.24 |
| " # 9 | 3.5 | 12.25 |
| " #10 | 13.6 | 184.96 |

$\Sigma y_i = \underline{92.0}$ $\Sigma y_i^2 = \underline{1153.58}$

(3)--- $\sum_{i=1}^{n} y_i = \sum_{i=1}^{10} y_i = y_1 + y_2 + y_3 + \cdots + y_{10} = \underline{92.0}$ recorded as shown above.

(4)---The mean, or average: $\bar{y} = \frac{\Sigma y_i}{n} = \frac{92.0}{10} = 9.20$ hundreds of ft. <u>or 920 ft</u>.

(5)---Squaring each measurement:

$y_1^2 = y_1 \cdot y_1 = (.5)\cdot(.5) = .25$ recorded as shown.

$y_2^2 = y_2 \cdot y_2 = (21.3)\cdot(21.3) = 453.69$ " " "

$\vdots \quad \vdots \quad \vdots$

$y_{10}^2 = y_{10} \cdot y_{10} = (13.6)\cdot(13.6) = 184.96$ " " "

(6)---The sum of the squares: $\Sigma y_i^2 = y_1^2 + y_2^2 + \cdots + y_{10}^2 =$

$.25 + 453.69 + \cdots + 184.96 = \underline{1153.58}$ recorded as shown.

(7)---The variance $s^2$:

$$s^2 = \frac{n(\Sigma y_i^2) - (\Sigma y_i)^2}{n(n-1)} = \frac{10(1153.58) - (92.0)^2}{10(10-1)} =$$

$$\frac{11535.8 - (92.0)\cdot(92.0)}{10(9)} = \frac{11535.8 - 8464}{90} =$$

$$\frac{3071.8}{90} = \underline{34.13}$$

(8)---The Standard Deviation s:

$$s = \sqrt{s^2} = \sqrt{34.13} = 5.84 \text{ hundreds of ft. } \underline{\text{or } 584 \text{ ft}}.$$

GUIDESHEET #2. The Time-Plot

Steps:

1---Obtain some graph-paper. It is much too tedious to draw a useful time-line on plain paper.

2---The graph-paper may be used vertically or horizontally, whichever allows the necessary number of divisions to be used for each axis. Consider the inconvenience to the reader if he has to turn a bound paper or report sideways to read the graph.

3---For a time-plot, the vertical axis is used for measurements and the horizontal axis for time-period numbers. If measurements cover a short range of high values, you may wish to start the scale on the vertical axis just below the lowest possible measurement value and end just above the highest. If this is done, draw in a discontinuity just above where the two axes cross (the "origin" or zero point for each axis) to indicate this was done.

4---Label the vertical axis the y-axis. Also write in "scores" or "measurements" or "CEPs" in parentheses, whatever is appropriate. Label every <u>bold</u> horizontal line with an even number in your range of values.

5---Label the horizontal axis the x-axis. This axis is for the time-period numbers. Time-period numbers (1, 2, •••, n) are used rather than the time-period names to aid in computation and to avoid clutter and confusion. For instance, tests may not be given, or flights flown, on the same date each time-period, so trying to record day and month would get messy. Your table has the correct information, presumably.

6---Leave plenty of time-periods in the future for trending and forecasting.

7---Plot the first measurement. That is, for x=1, plot the corresponding y-value. For trending, there should only be one measurement plotted for each time-period. If there are more than one for any single time-period, the mean or average is the appropriate value to plot. See Guidesheet #1.

8---If there is a missing measurement, do not count that time-period at all, that is, no time-period number is assigned to that month, for instance.

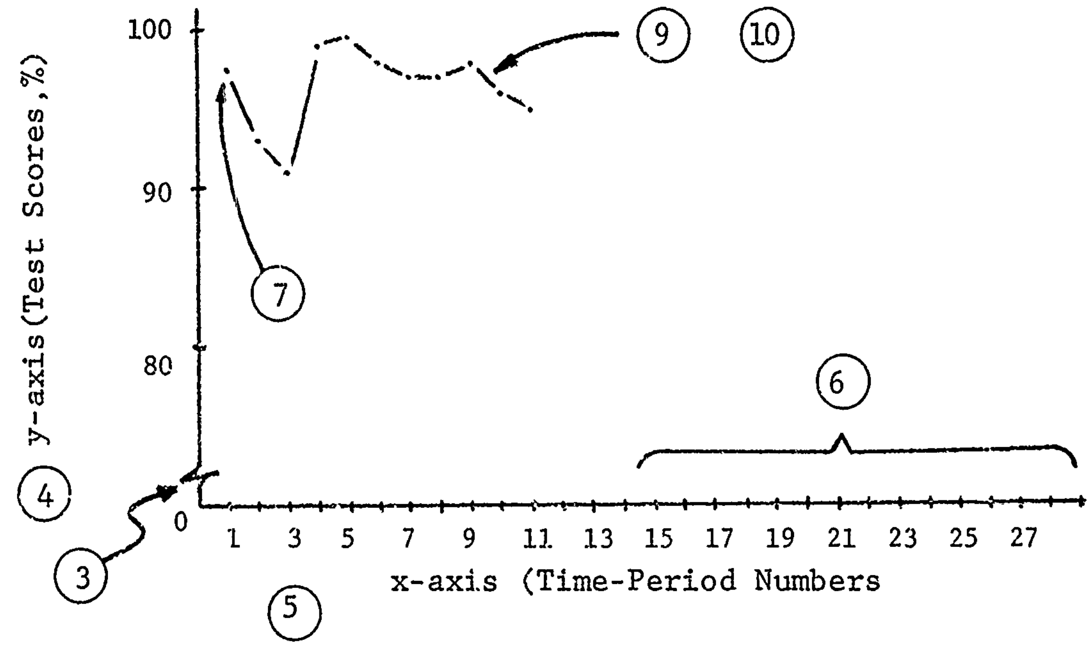
9---Plot the remaining values.

10--Connect each point with short line-segments. This will give you an idea about what sort of trend there is, thus which trending method would be most appropriate. Do this in pencil, because it should be erased before superimposing the computed trend line.

Notes:

1---If text is to be added and it must accompany the graph, graph-paper is reproducible, so xerox it, cut it out, and place it above your text. If necessary, reproduce the combined page to avoid glueing mess.

EXAMPLE for Time-Plot (Guidesheet #2)




Steps 1 and 2 are self explanatory. The remaining step-numbers are placed at the appropriate place.

| Time-Period Name | Time-Period Number $x_i$ | Measurement (Test Scores, %) $y_i$ |
|---|---|---|
| 9 Jan | 1 | 97.6 |
| 3 Feb | 2 | 93.0 |
| 4 Mar | 3 | 91.0 |
| 28 Apr | 4 | 99.0 |
| 1 May | 5 | 99.5 |
| 6 Jun | 6 | 98.0 |
| 5 Jul | 7 | 97.0 |
| ~~3 Aug~~ | | Missing, No Test |
| 22 Sept | 8 | 97.0 |
| 16 Oct | 9 | 98.0 |
| 17 Nov | 10 | 96.0 |
| 8 Dec | 11 | 95.0 |

8

GUIDESHEET #3. The Control Chart for Small Samples

Assumptions:

1---The measurements are a sample from a normal population. That is, if there were many of these measurements, they would be distributed according to the well-known "bell-curve". Usually measurements such as test scores, bomb scores, etc., can be treated as normally distributed. Another way of checking this is to ask if there were many measurements of this type, would most be near the average and few far from it, both above and below. If the answer is yes, this assumption is probably a good one. If this assumption cannot be met, go to Guidesheet #4.

2---The sample drawn is a random sample, not biased.

Steps:

1---Assume a value for the population mean $\mu=\mu_o$. This may be the actual population mean (rarely known for sure) or a hypothesized mean such as a standard one wishes to meet or continue to meet.

2---Draw a chart for your measurements (y-axis) over time (x-axis). Draw a horizontal line at $y=\mu_o$.

3---Count the number of measurements that make up the sample each time-period. This is the sample size "n".

4---Compute the sample standard deviation "s" (see Guidesheet #1). This s should be nearly constant across all time-periods. If it is not, see Note #3.

5---Choose a level of significance $\alpha$.

6---Look up the table value of the t-statistic (Table I on page 62) corresponding to the chosen $\alpha$ and degrees-of-freedom $\nu=n-1$. Call this value $t(\text{Table})=t_\alpha$.

(7)---Compute the Lower Control Limit (LCL) of the Control Chart according to the following formula:

$$LCL = \mu_o - t(\text{Table}) \cdot \frac{s}{\sqrt{n}} \qquad (1)$$

(8)---Draw a horizontal line at y=LCL on the Control Chart. A dotted line or different color line may be used to distinguish it from the line $y=\mu_o$.

(9)---If any single sample mean ever falls below the LCL, or means begin to approach it (a trend), then this indicates that, with probability ($1-\alpha$), you are no longer meeting your standard, based on the samples taken. That is, a trend line, described by one of the methods in this guidebook, is <u>significant</u> at the $\alpha$-level if it touches the LCL.

Notes:

(1)---If the UCL (Upper Control Limit) is needed instead,

$UCL=\mu_o+t(\text{Table})\cdot\frac{s}{\sqrt{n}}$ . All else is the same.

(2)---The <u>smaller</u> one chooses $\alpha$ to be, the <u>further</u> from the standard the control limit line (UCL or LCL) will be. A <u>very</u> small $\alpha$, however, may limit your ability to detect trends.

(3)---If the standard deviation s changes significantly over the time-periods, the control limit must be recomputed and replotted <u>each time-period</u>. This would make a segmented limit. It does <u>not</u> invalidate the method in any way (Ref 4:426). If this is not desirable, a non-parametric method (Guidesheet #11, 12, or 13) must be used.

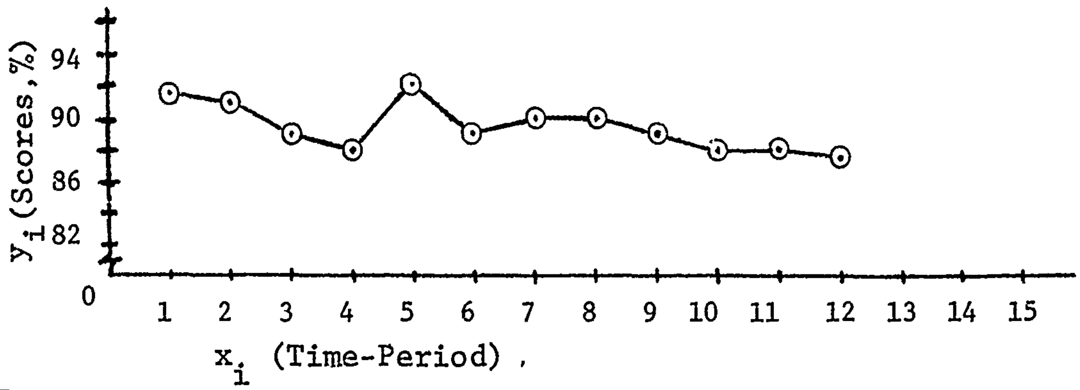
EXAMPLE for Control Chart for Small Samples (Guidesheet #3)

Steps:

1 ---The Wing Commander is interested in how well his people will measure up to a standard of 90% correct responses on Stan/Eval tests in the next six months. He asks his trend analysis shop to monitor Stan/Eval test scores for trends. Our Wing Commander is assuming $\mu_o$=90%.

2 ---The Wing is large and only a few personnel are tested each month, actually six. The TA officer knows it will be helpful to look at past records at least as a start and to see if the 90% standard is a reasonable one. He draws a chart covering all past data plus plenty of room for forecasting or recording new data. The data and chart are shown here:

| Month | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | #10 | #11 | #12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Score | 91.5 | 91 | 89 | 88 | 92.1 | 90 | 90 | 89 | 89 | 88 | 88 | 87.5 |




3 ---Records showed that about the same number of persons are tested each month and the scores don't seem to vary a great deal, unless there is a big turnover in people. That is, n=6 every month.

4 ---The computed sample standard deviation (Guidesheet #1) turned out to be s=3. The quantity s was nearly constant over time.

(5)---The TA officer wanted to be reasonably sure he did not report a trend when there was actually none (Type I error), his heart being with the crew members, so he picked $\alpha$=.01.

(6)---Looking in Table I, he found t(Table) to be 3.365. For $\alpha$=.01, and $\nu$=n-1=5, t(Table)=3.365. See portion of Table below.

| $\nu$ | $\alpha$ = 0.10 | $\alpha$ = 0.05 | $\alpha$ = 0.025 | $\alpha$ = 0.01 | $\alpha$ = |
|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5 |
| 4 | 1.533 | 2.132 | 2.776 | 3.474 | 4 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3 |

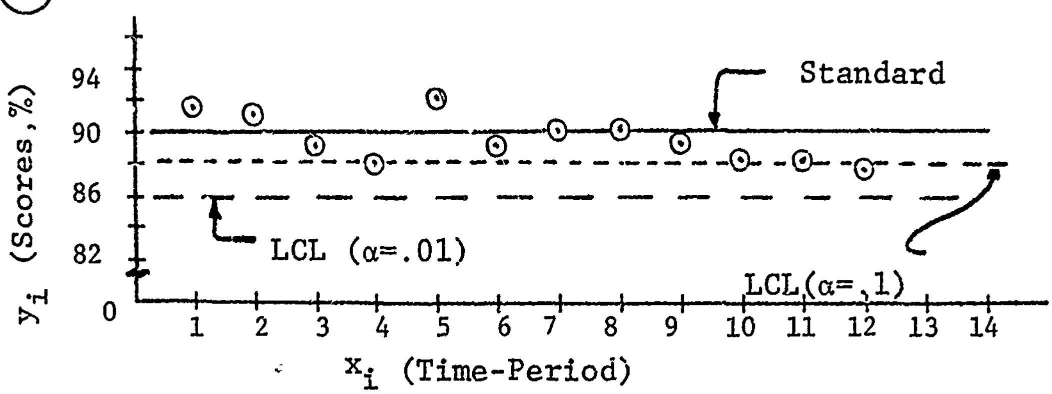
(7)---The lower control limit was computed as follows:

$$LCL = \mu_o - t(\text{Table}) \cdot \frac{s}{\sqrt{n}}$$

$$= 90-(3.365)\cdot(1.225) = 90-4.12 = 85.88$$

LCL = <u>85.9</u>

(8)---The control chart was drawn as shown.




(9)---If $\alpha$ were not so small, say $\alpha$=.10, LCL would be 88.2 One is less sure, however, when he says there is a trend. He has an $\alpha$=.10 probability of saying there is a trend when there is none as opposed to $\alpha$=.01 (<u>much</u> less probability).

# Table I
## The Student-T Distribution

Values of $t_\alpha$*

| $\nu$ | $\alpha = 0.10$ | $\alpha = 0.05$ | $\alpha = 0.025$ | $\alpha = 0.01$ | $\alpha = 0.005$ | $\nu$ |
|---|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 |
| 4 | 1.533 | 2.132 | 2.776 | 3.474 | 4.604 | 4 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 |
| inf | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | inf. |

GUIDESHEET #4. The Control Chart for Large Samples

Assumptions:

1---Guidesheet #3 cannot be used because the assumption of normality is not a reasonable assumption. However, to use this method, "n" (number of measurements in a sample) must be large, $n>30$. If $n\leq30$ and we cannot assume normality, then non-parametric methods must be used to determine significance of trends. See Guidesheet #11, 12, or 13.

2---The sample drawn is a random sample, not biased.

Steps:

1---Assume a value for the population mean $\mu=\mu_o$. This may be the actual population mean (rarely known for sure) or a hypothesized mean such as a standard one wishes to meet or continue to meet.

2---Draw a chart for your measurements (y-axis) over time (x-axis). Draw a horizontal line at $y=\mu_o$.

3---Count the number of measurements that make up the sample each time-period. This is the sample size "n".

4---Compute the sample standard deviation "s" (see Guidesheet #1). This s should be nearly constant across all time-periods. If it is not, see Note #3.

5---Choose a level of significance $\alpha$.

6---Look up the table value of the z-statistic in Note #4 corresponding to the chosen value of $\alpha$. Call this value z(Table)=$z_\alpha$.

7---Compute the Lower Control Limit (LCL) of the Control Chart according to the following formula:

$$LCL = \mu_o - z(\text{Table}) \cdot \frac{s}{\sqrt{n}} \qquad (1)$$

8---Draw a horizontal line at y=LCL on the Control Chart. A dotted line or different color line may be used to distinguish it from the line $y=\mu_o$.

9---If any single sample mean ever falls below the LCL, or means begin to approach it (a trend), then this indicates that, with probability $(1-\alpha)$, you are no longer meeting your standard, based on the samples taken. That is, a trend line, described by one of the methods in this guidebook, is significant at the $\alpha$-level if it touches the LCL.

Notes:

1---If the UCL (Upper Control Limit) is needed instead, $UCL=\mu_o+z(\text{Table})\cdot\frac{s}{\sqrt{n}}$ . All else is the same.

2---The smaller one chooses $\alpha$ to be, the further from the standard the control limit line (UCL or LCL) will be. A very small $\alpha$, however, may limit your ability to detect trends.

3---If the standard deviation s changes significantly over the time-periods, the control limit must be recomputed and replotted each time-period. This would make a segmented limit. It does not invalidate the method in any way (Ref 4:426). If this is not desirable, a non-parametric method (Guidesheet #11, 12, or 13) must be used.

4---Values of z-statistic for various values of $\alpha$ are given here

| | | | | |
|---|---|---|---|---|
| $\alpha$ = 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| z = 1.28 | 1.645 | 1.96 | 2.33 | 2.575 |

EXAMPLE for Control Chart for Large Samples (Guidesheet #4)

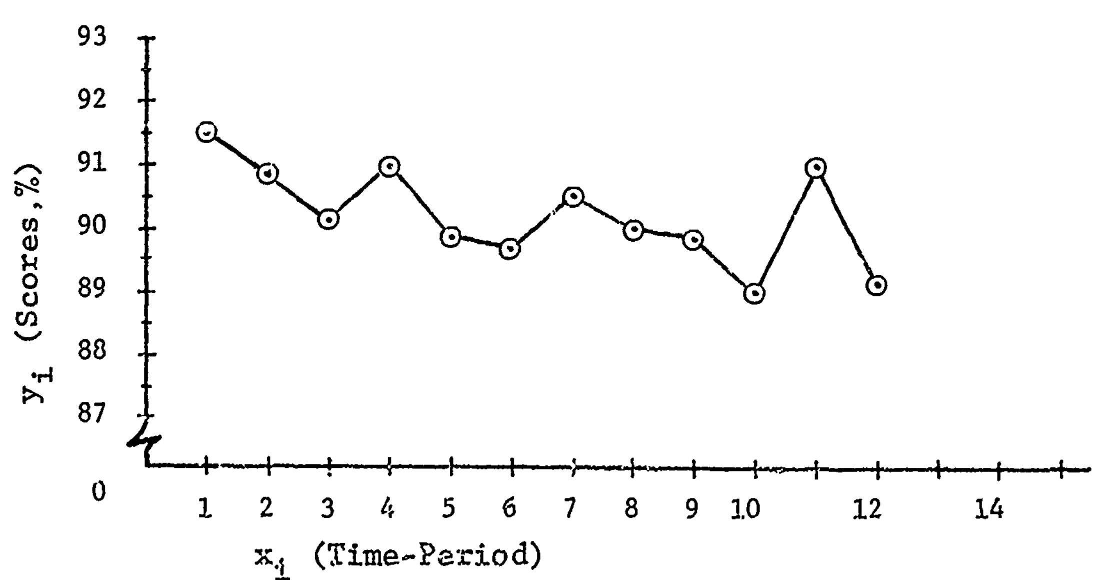
Steps:

(1)---The trend analysis officer at Headquarters keeps track of how well his command does on Stan/Eval tests each period. Since he requires nearly the same number of tests to be reported each period, and this number is greater than 30, he chooses this method (large-sample) to build a control chart. He is interested in maintaining a standard of 90% correct responses on tests so his hypothesized mean is $\mu_o$=90%.

(2)---The TA officer has past data, so he plots it over time for informational purposes as shown and draws in $y=\mu_o$:

| Time-Period: | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 |
|---|---|---|---|---|---|---|---|---|
| Average Score: | 91.5 | 90.8 | 90.2 | 91 | 89.8 | 89.7 | 90.5 | 90 |

| #9 | #10 | #11 | #12 |
|---|---|---|---|
| 89.8 | 89 | 91 | 89.2 |




(3)---The number of test scores turned in each time-period is around 31, so he uses n=31.

(4)---The sample standard deviation s is computed according to Guidesheet #1 and is equal to s=3.3. Records showed this was nearly constant in the past and there was no reason to believe it would not stay nearly constant.

(5)---The TA officer chooses $\alpha=.01$, thus keeping the probability of a Type I error low. That is, he does not care to report a trend when there's a good chance there is none.

(6)---From the table in Note #4 we get the table value of the test-statistic corresponding to $\alpha=.01$:

$$z(\text{Table}) = 2.33$$

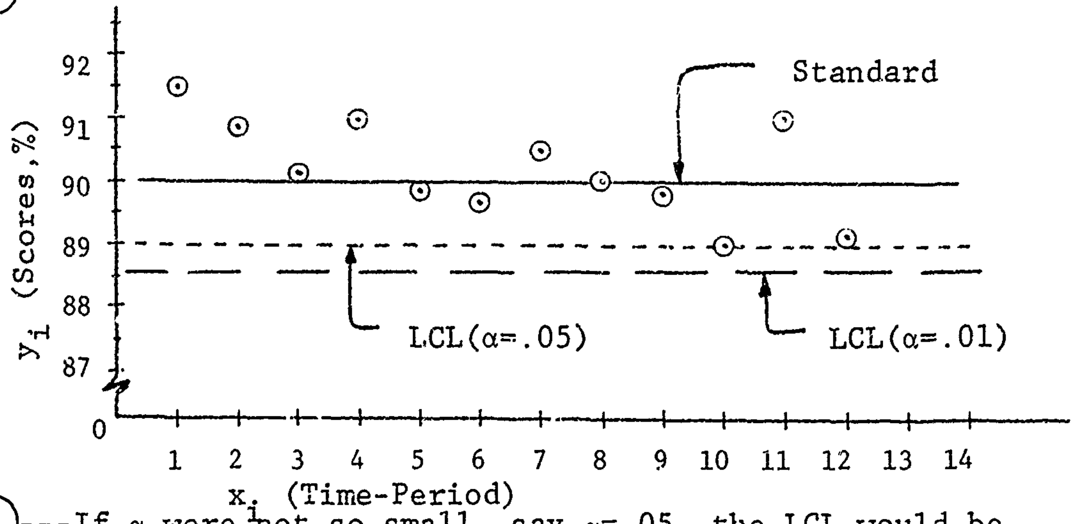
(7)---The Lower Control Limit (LCL) is computed as shown:

$$\text{LCL} = \mu_o - z(\text{Table}) \cdot \frac{s}{\sqrt{n}}$$

$$= 90 - 2.33\left(\frac{3.3}{\sqrt{31}}\right) = 90 - 2.33(.593) = 88.618$$

$$\text{LCL} = \underline{88.6}$$

(8)---The TA officer adds the LCL to the chart as shown:




(9)---If $\alpha$ were not so small, say $\alpha=.05$, the LCL would be 89.0%. One would be less sure, however, when he

says there is a trend. He has a $\alpha = .05$ probability of saying there is a trend when there actually is none (as opposed to a probability of .01).

GUIDESHEET #5. Straight-Line Least-Squares-Best-Fit.

Assumptions:

1---This method is to be used when the time-plot of the data suggests a straight-line trend.

Steps:

1---Arrange the data in vertical columns (as in example) with column headings as follows. Columns 4 and 5 will be filled in later.

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Square $x_i^2$ | Product $x_i \cdot y_i$ |
|---|---|---|---|---|
| | | | | |
| | $\Sigma x_i =$ ____ | $\Sigma y_i =$ ____ | $\Sigma x_i^2 =$ ____ | $\Sigma x_i \cdot y_i =$ ____ |

2---Count the total number of observations or measurements that are to be tested for trend. Call this total number "n".

3---Compute the square of each period number. That is, compute $x_i^2 = x_i \cdot x_i$ and record each value in the designated column.

4---Compute the product of each period number and measurement. That is, compute the product $x_i \cdot y_i$ and record in the designated column.

5---Add up each column. That is, compute the sums: $\Sigma x_i$, $\Sigma y_i$, $\Sigma x_i^2$, and $\Sigma x_i \cdot y_i$ and record in the appropriate place at the bottom of each column.

6---Compute the Mean of $x_i$ and Mean of $y_i$ according to the following formulas.

$$\bar{x} = \frac{\Sigma x_i}{n} \qquad (1)$$

$$\bar{y} = \frac{\Sigma y_i}{n} \tag{2}$$

7---Compute the quantity $S_{xx}$.

$$S_{xx} = n\sum x_i^2 - (\sum x_i)^2 \tag{3}$$

8---Compute the quantity $S_{xy}$.

$$S_{xy} = n\sum x_i y_i - (\sum x_i)(\sum y_i) \tag{4}$$

9---Compute "b", the "slope" of the line-of-best-fit.

$$b = \frac{S_{xy}}{S_{xx}} = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2} \tag{5}$$

10--Compute "a", the y-intercept of the line-of-best-fit.

$$a = \bar{y} - b\bar{x} \tag{6}$$

11--Write the algebraic equation of the line-of-best-fit.

$$y = a + bx \tag{7}$$

12--Draw the line-of-best-fit through your data points. Two points allow you to do this. At year 0 (x=0) the equation says y=a so the line must pass through this point. Substitute any other value of x (preferably near the end of the data) and plot the corresponding value of y. Draw a straight line through these points.

13--When used in conjunction with a control chart, a trend becomes significant when the trend line touches one of the limits, either the LCL or the UCL. See Guidesheet #3 or #4.

Notes:

1---If the slope of the line, b, is zero, there is no

trend. This would be a quick check. That is, the size of "b" can be looked at as an indication of the magnitude of the trend.

EXAMPLE For Straight-Line Least-Squares-Best-Fit
(Guidesheet #5)

Steps:

1---

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Square $x_i^2$ | Product $x_i \cdot y_i$ |
|---|---|---|---|---|
| Jan | 1 | 97.6 | 1 | 97.6 |
| Feb | 2 | 93.0 | 4 | 186.0 |
| Mar | 3 (1) | 91.0 | 9 | 273.0 |
| Apr | 4 (2) | 99.0 | 16 | 396.0 |
| May | 5 (3) | 99.5 | 25 | 497.5 |
| Jun | 6 (4) | 98.0 | 36 | 588.0 |
| Jul | 7 (5) | 97.0 | 49 | 679.0 |
| Aug | 8 (6) | 96.0 | 64 | 768.0 |
| Sept | 9 (7) | 97.0 | 81 | 873.0 |
| Oct | 10 (8) | 98.0 | 100 | 980.0 |
| Nov | 11 (9) | 96.0 | 121 | 1056.0 |
| Dec | 12 (10) | 95.0 | 144 | 1140.0 |
| | $\Sigma x_i = \underline{78}$ | $\Sigma y_i = \underline{1157.1}$ | $\Sigma x_i^2 = \underline{650}$ | $\Sigma x_i \cdot y_i = \underline{7534.1}$ |

2---n=12. Note: If you wished not to include the first two measurements in this test for trend due to their position in the time-line, you would have to re-number both time-line and table from 1 to 10 (shown in parentheses). This is <u>not done</u> here.

3---Compute squares $x_i^2$:

$x_1^2 = x_1 \cdot x_1 = 1 \cdot 1 = 1$ recorded as shown.

$x_2^2 = x_2 \cdot x_2 = 2 \cdot 2 = 4$ " " "

$x_3^2 = x_3 \cdot x_3 = 3 \cdot 3 = 9$ " " "

$\vdots$ " " "

$x_{12}^2 = x_{12} \cdot x_{12} = 12 \cdot 12 = 144$ " " "

4---Compute products $x_i \cdot y_i$:

$x_1 \cdot y_1 = 1 \cdot 97.6 = 97.6$ recorded as shown.

$x_2 \cdot y_2 = 2 \cdot 93.0 = 186.0$ " " "

$x_3 \cdot y_3 = 3 \cdot 91.0 = 273.0$ " " "

$\vdots$ " " "

$x_{12} \cdot y_{12} = 12 \cdot 95.0 = 1140.0$ " " "

5---Sum

$\Sigma$ = 78 recorded as shown

Su

$\Sigma$ 5.0 = 1157.1 recorded as shown

Sum column 4.

$\Sigma x_i^2 = 1+4+9+16+\cdots+144 = 650.0$ recorded as shown

Sum column 5:

$\Sigma x_i y_i = 97.6+186+273+\cdots+1140 = 7534.1$ recorded as shown.

6---Compute mean of $x_i$ and $y_i$:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{78}{12} = 6.5 \qquad \bar{x} = 6.5$$

$$\bar{y} = \frac{\Sigma y_i}{n} = \frac{1157.1}{12} = 96.425 \qquad \bar{y} = 96.425$$

7---Compute $S_{xx}$:

$$S_{xx} = n\sum x_i^2 - (\sum x_i)^2 = 12(650)-(78)^2$$

$$=7800 - 6084 = 1716$$

$$S_{xx} = 1716$$

(8)---Compute $S_{xy}$:

$$S_{xy} = n\sum x_i \cdot y_i - (\sum x_i)(\sum y_i)$$

$$= 12(7534.1) - (78)(1157.1)$$

$$= 90{,}409.2 - 90{,}253.8 = 155.4$$

$$S_{xy} = 155.4$$

(9)---Compute the slope "b":

$$b = \frac{S_{xy}}{S_{xx}} = \frac{155.4}{1716} = .0906$$

$$b = .0906$$

(10)--Compute "a", the y-intercept:

$$a = \bar{y} - b\bar{x} = 96.425 - (.0906)(6.5)$$

$$= 96.425 - .588$$

$$a = 95.8$$

(11)--Write the equation of the line: $y = 95.8 + .0906x_i$

(12)--Plot the line:

$y=95.8+.0906x_i$

If $x=0$, $y=95.8$

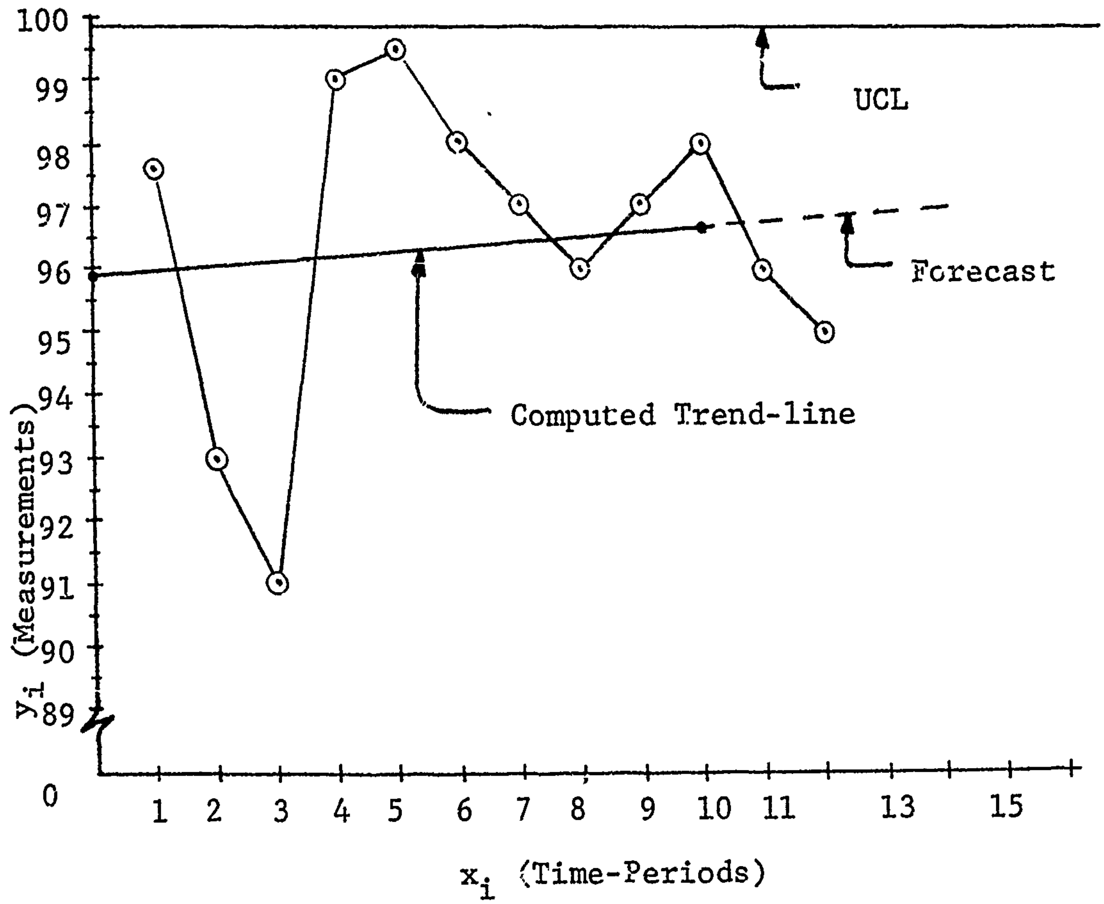
If $x=10$, $y=95.8+.906$

$y=96.7$

Note: If "b" had been a negative number, the line of best-fit would have had a downward slope.

(13)--If for some given $\alpha$, a UCL was computed to be UCL= 98.0, and plotted as shown, one could see the trend line does not touch for many time-periods. That is,

there would be plenty of chance for measurements to reverse direction or level off. This illustrates Caution #1, page 50, that one should reserve judgement unless trends reach a limit before one could observe new measurements.



GUIDESHEET #6. Curvilinear Least-Squares-Best-Fit

Assumptions:

(1)---This method is to be used when a time-plot of the data, suggests a curved-line trend.

Steps:

(1)---Arrange the data in vertical columns (as in example) with column headings as follows. Columns 4 through 8 will be filled in later.

| Period Name | Period # $x_i$ | Measurement $y_i$ | Square $x_i^2$ | Cube $x_i^3$ | Fourth $x_i^4$ | Product $x_i \cdot y_i$ | Product $x_i^2 \cdot y_i$ |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | $\Sigma x_i$ | $\Sigma y_i$ | $\Sigma x_i^2$ | $\Sigma x_i^3$ | $\Sigma x_i^4$ | $\Sigma x_i \cdot y_i$ | $\Sigma x_i^2 \cdot y_i$ |
| | =____ | =____ | =____ | =____ | =____ | =____ | =____ |

(2)---Count the total number of observations or measurements which are to be tested for trend. Call this total number "n".

(3)---Compute the square of each period number $x_i$ and record these $x_i^2$ values in column 4.

(4)---Compute the cube of each period number $x_i$. This may be done by multiplying the $x_i^2$ values in column 4 by $x_i$, $x_i^3 = x_i^2 \cdot x_i$. Record each $x_i^3$ value in column 5.

(5)---Compute the fourth power of each $x_i$. This may be done by multiplying the $x_i^3$ values in column 5 by $x_i$, or $x_i^4 = x_i^3 \cdot x_i$. Record each $x_i^4$ value in column 6.

(6)---Multiply each $x_i$ and $y_i$ and record each product in column 7.

(7)---Multiply each quantity $x_i \cdot y_i$ by $x_i$. That is, compute

each product $x_i^2 \cdot y_i$. Record each $x_i^2 \cdot y_i$ value in column 8.

(8)---Sum each column $\Sigma x_i$, $\Sigma y_i$, $\Sigma x_i^2$, $\Sigma x_i^3$, $\Sigma x_i^4$, $\Sigma x_i \cdot y_i$, and $\Sigma x_i^2 \cdot y_i$. Record these sums in the appropriate place below each column.

(9)---Arrange three equations as shown below. The values in parentheses have been computed in previous steps and are the "coefficients" of the unknowns a, b, and c. We will be solving this system of three simultaneous equations for the three unknowns a, b, and c. Number the equations (1), (2), and (3) as shown.

$$(n)a + (\Sigma x)b + (\Sigma x^2)c = (\Sigma y) \qquad (1)$$

$$(\Sigma x)a + (\Sigma x^2)b + (\Sigma x^3)c = (\Sigma xy) \qquad (2)$$

$$(\Sigma x^2)a + (\Sigma x^3)b + (\Sigma x^4)c = (\Sigma x^2 y) \qquad (3)$$

(10)--Eliminate the unknown "a" between equations (1) and (2) See Note 1. To do this, multiply equation (1) by the coefficient of "a" in equation (2) to form equation (4). Multiply equation (2) by the coefficient of "a" in equation (1) to form equation (5). Subtract equation (5) from equation (4) to form equation (6), with the unknown "a" eliminated.

(11)--Eliminate the same unknown "a" between equations (1) and (3) by the same method as Step 10. Multiply (1) by the coefficient of "a" in (3) to form equation (7). Multiply (3) by the coefficient of "a" in (1) to form equation (8). Subtract (8) from (7) to form equation (9), with the unknown "a" eliminated.

(12)--Eliminate the unknown "b" between equations (6) and (9). Multiply (6) by the coefficient of "b" in (9) to form equation (10). Multiply (9) by the coefficient of "b" in (6) to form equation (11). Subtract (11) from (10) to form equation (12) with the unknown "b" eliminated.

(13) --Equation (12) can be readily solved yielding the correct value of "c". That is, dividing the number on the right-hand-side of the equals sign in equation (12) by the coefficient of "c", yields the value "c".

(14) --Substitute the value of "c" from Step 13 into equation (6) and solve for the unknown "b" such as was done in Step 13.

(15) --Substitute the values of "b" and "c" into equation (1) and solve for the unknown "a".

(16) --Substitute all three values "a", "b", and "c" into either equation (2) or (3) to check the answers. If the equation does not yield an equality, go back and check each step. This completes the solution of the set of simultaneous equations.

(17) --The equation of the least-squares-best-fit parabola can be written with the values "a", "b", and "c" from earlier steps as follows:

$$y = a + b \cdot x + c \cdot x^2 \qquad (13)$$

(18) --To draw the curve of best-fit one will have to plot several of the points of the parabola described by equation (13). Substitution of several integer values of $x_i$ ($x_i$=1, 2, 3, etc.) will yield several values of $y_i$. When these are plotted, a French Curve may be used to draw a line connecting them. One may wish to use a different color of pencil for the points and line if plotting on the same time-plot that contains the original data.

(19) --The trend described by equation (13) becomes significant at the $\alpha$-level of significance (see Step 6, Guidesheet #3 or #4) when it touches the appropriate Control-Chart limit (UCL or LCL).

Notes:

1---Sometimes it is apparent by inspection which unknowns should be eliminated first, making the numbers work out better. Coefficients being multiples of each other or even negatives of each other simplify elimination. A little practice solving sets of equations provide one with the ability to see such short-cuts but may not be assumed common knowledge in this general solution guide.

2---When a new measurement is taken, for the next time-period, a new equation describing the trend must be computed. That is, all the steps must be followed again. This is not necessary, however, if the new measurement falls on or very close to the trend line.

EXAMPLE for Curvilinear Least-Squares-Best-Fit
(Guidesheet #6)

Steps:

1---The data is arranged as shown:

| Period Name (Half-Yr) | Period # $x_i$ | Measurement $y_i$ | Square $x_i^2$ | Cube $x_i^3$ | Fourth $x_i^4$ | Product $x_i \cdot y_i$ | Product $x_i^2 \cdot y_i$ |
|---|---|---|---|---|---|---|---|
| Jul-Dec '71 | 1 | 18.70 | | | | | |
| 1st '72 | 2 | 14.90 | | | | | |
| 2nd '72 | 3 | 6.70 | | | | | |
| 1st '73 | 4 | 5.73 | | | | | |
| 2nd '73 | 5 | 9.35 | | | | | |
| 1st '74 | 6 1 | 21.20 | 1 | 1 | 1 | 21.2 | 21.2 |
| 2nd '74 | 7 2 | 20.60 | 4 | 8 | 16 | 41.2 | 82.4 |
| 1st '75 | 8 3 | 23.30 | 9 | 27 | 81 | 69.9 | 209.7 |
| 2nd '75 | 9 4 | 23.30 | 16 | 64 | 256 | 93.2 | 372.8 |
| 1st '76 | 10 5 | 27.40 | 25 | 125 | 625 | 137.0 | 685.0 |
| 2nd '76 | 11 6 | 30.20 | 36 | 216 | 1296 | 181.2 | 1087.2 |
| 1st '77 | 12 7 | 28.90 | 49 | 343 | 2401 | 202.3 | 1461.1 |
| 2nd '77 | 13 8 | 29.60 | 64 | 512 | 4096 | 236.8 | 1894.4 |
| 1st '78 | 14 9 | 29.00 | 81 | 729 | 6561 | 261.0 | 2349.0 |
| | $\Sigma x_i$ =45 | $\Sigma y_i$ =233.5 | $\Sigma x_i^2$ =285 | $\Sigma x_i^3$ =2025 | $\Sigma x_i^4$ =15333 | $\Sigma x_i \cdot y_i$ =1243.8 | $\Sigma x_i^2 \cdot y_i$ =8117.8 |

2---It was determined that <u>not all</u> the above data should enter into the test for trend due to a "rules change" in the last half of 1973 ($x_i$=5). The table above <u>and</u> time-plot is renumbered accordingly.

3---Compute each square $x_i^2$:

$x^2 = x \cdot x = 1 \cdot 1 = 1$ Record in Column #4

$x_2^2 = x_2 \cdot x_2 = 2 \cdot 2 = 4$ " " " "

$x_3^2 = x_3 \cdot x_3 = 3 \cdot 3 = 9$ " " " "

. . . . " " " "

. . . . " " " "

. . . . " " " "

$x_9^2 = x_9 \cdot x_9 = 9 \cdot 9 = 81$ " " " "

(4)---Compute each cube $x_i^3$:

$$
\begin{array}{lllll}
x_1^3 = x_1^2 \cdot x_1 = & 1 \cdot 1 = & 1 & \text{Record in Column \#5} \\
x_2^3 = x_2^2 \cdot x_2 = & 4 \cdot 2 = & 8 & \text{" " " "} \\
\vdots & \vdots & \vdots & \text{" " " "} \\
x_9^3 = x_9^2 \cdot x_9 = & 81 \cdot 9 = & 729 & \text{" " " "}
\end{array}
$$

(5)---Compute the fourth power $x_i^4$:

$$
\begin{array}{lllll}
x_1^4 = x_1^3 \cdot x_1 = & 1 \cdot 1 = & 1 & \text{Record in Column \#6} \\
x_2^4 = x_2^3 \cdot x_2 = & 8 \cdot 2 = & 16 & \text{" " " "} \\
\vdots & \vdots & \vdots & \text{" " " "} \\
x_9^4 = x_9^3 \cdot x_9 = & 729 \cdot 9 = & 6561 & \text{" " " "}
\end{array}
$$

(6)---Compute the product $x_i \cdot y_i$:

$$
\begin{array}{llll}
x_1 \cdot y_1 = & 1 \cdot 21.2 = & 21.2 & \text{Record in Column \#7} \\
x_2 \cdot y_2 = & 2 \cdot 20.6 = & 41.2 & \text{" " " "} \\
\vdots & \vdots & \vdots & \text{" " " "} \\
x_9 \cdot y_9 = & 9 \cdot 29.0 = & 261.0 & \text{" " " "}
\end{array}
$$

(7)---Compute the product $x_i^2 \cdot y_i$:

$$
\begin{array}{llll}
x_1^2 \cdot y_1 = & 1 \cdot 21.2 = & 21.2 & \text{Record in Column \#8} \\
x_2^2 \cdot y_2 = & 4 \cdot 20.6 = & 82.4 & \text{" " " "} \\
\vdots & \vdots & \vdots & \text{" " " "} \\
x_9^2 \cdot y_9 = & 81 \cdot 29.0 = & 2349.0 & \text{" " " "}
\end{array}
$$

(8)---Sum the $x_i$'s:

$$\Sigma x_i = x_1 + x_2 + \cdots + x_9 = 1 + 2 + \cdots + 9 = \underline{45}$$

Record in Column #2

Sum the $y_i$'s:

$$\Sigma y_i = y_1+y_2+\cdots+y_9 = 21.2+20.6+\cdots+29.0 = \underline{233.5}$$

Record in Column #2

Sum the remaining columns and record above.

$$\Sigma x_i^2 = x_1^2+x_2^2+\cdots+x_9^2 = 1+4+\cdots+81 = \underline{285}$$

$$\Sigma x_i^3 = x_1^3+x_2^3+\cdots+x_9^3 = 1+8+\cdots+729 = \underline{2025}$$

$$\Sigma x_i^4 = x_1^4+x_2^4+\cdots+x_9^4 = 1+16+\cdots+6561 = \underline{15,333}$$

$$\Sigma x_i \cdot y_i = x_1y_1+x_2y_2+\cdots+x_9y_9 = 21.2+41.2+\cdots+261.0 = \underline{1243.8}$$

$$\Sigma x_i^2\, y_i = x_1^2y_1+x_2^2y_2+\cdots+x_9^2y_9 = 21.2+82.4+\cdots+2349 = \underline{8117.8}$$

9 --- $9a + 45b + 285c = 233.5$ (1)

$45a + 285b + 2025c = 1243.8$ (2)

$285a + 2025b + 15,333c = 8117.8$ (3)

10 --Multiply (1) by 45:

$405a + 2025b + 12825c = 10507.5$ (4)

Multiply (2) by 9:

$405a + 2565b + 18225c = 11194.2$ (5)

Subtract (5) from (4) to get:

$0 - 540b - 5400c = -686.7$ (6)

11 --Multiply (1) by 285:

$2565 + 12825b + 81,225 = 66,547.5$ (7)

Multiply (3) by 9:

$2565 + 18225b + 137,997 = 73,060.2$ (8)

Subtract (8) from (7) to get:

$0 - 5400b - 56,772 = -6512.7$ (9)

12 --Note that the coefficient of b in Eq (9) is a multiple of the coefficient in (6). We may take advantage of this and come up with Eq (12) much easier. Then we will do it the long way (below) to show both are equivalent.

Multiply (6) by 10 to form (10):

| | | |
|---|---|---|
| | $-5400b - 54{,}000c = -6867$ | (10) |
| Recopy (9): | $-5400b - 56{,}772c = -6512.7$ | (11) |
| Subtract: 11 from 10 | $0 + 2772c = -354.3$ | (12) |

13 -- $2772c = -354.3$ (12)

Solving for c, we get:

$$c = \frac{-354.3}{2772} = -.128$$

$$c = \underline{-.128}$$

---

12 --Longer Method: If it is not recognized that coefficients of the unknowns are multiples, we have:

Multiply (6) by 5400:

$$2{,}916{,}000b - 29{,}160{,}000c = -3{,}708{,}180 \qquad (10)$$

Multiply (7) by 540:

$$2{,}916{,}000b - 30{,}656{,}880c = -3{,}516{,}858 \qquad (11)$$

Subtract (11) from (10) to get:

$$1{,}495{,}880c = -191{,}322 \qquad (12)$$

13 --Solving for c, we get:

$$c = \frac{-191{,}322}{1{,}495{,}800} = -.128$$

$c = \underline{-.128}$ Same Answer.

(14) --Substitute $c = -.128$ into Eq (6) and solve for "b":

$$-540b - 5400c = -686.7 \tag{6}$$

or

$$-540b - 5400(-.128) = -686.7$$

$$-540b + 691.2 = -686.7$$

$$-540b = -686.7 - 691.2 = -1377.9$$

$$b = \frac{-1377.9}{-540} = 2.55$$

$$b = \underline{2.55}$$

(15) --Substitute $c = -.128$ and $b = 2.55$ into Eq (1) and solve for "a":

$$9a + 45b + 285c = 233.5 \tag{1}$$

$$9a + 45(2.55) + 285\ (-.128) = 233.5$$

$$9a = 233.5 + 36.48 - 114.75 = 155.23$$

$$a = \frac{155.23}{9} = 17.2478$$

$$a = \underline{17.25}$$

(16) --Substitute these values of a, b, and c into Eq (2) to check the computations:

$$45a + 285b + 2025c = 1243.8 \tag{2}$$

$$45(17.25) + 285(2.55) + 2025(-.128) \stackrel{?}{=} 1243.8$$

$$776.25 + 726.75 - 259.2 \stackrel{?}{=} 1243.8$$

$$1243.8 \stackrel{?}{=} 1243.8$$

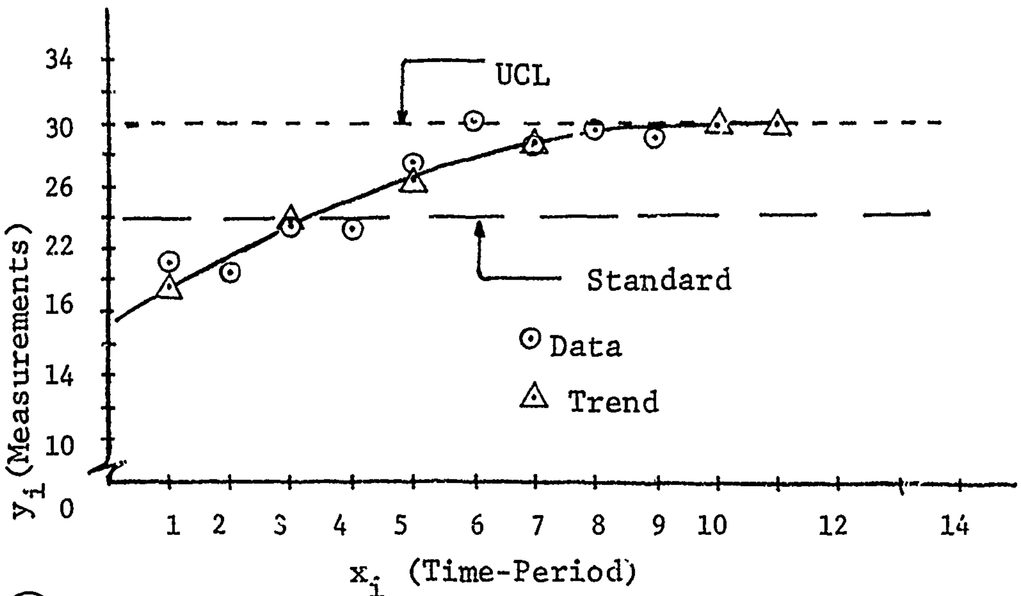
Yes, exactly. Computations are good.

(17) --The equation of best-fit curve is as follows:

$$y = 17.25 + 2.55x - .128x^2$$

(18) --Plotting points for $x = 1, 3, 5$, and $7$ should give us enough to draw the curve as shown below:

$$\text{If } x=1, \quad y = 17.25 + 2.55(1) - .128(1^2) = \underline{19.67}$$
$$x=3, \quad y = 17.25 + 2.55(3) - .128(3^2) = \underline{23.75}$$
$$x=5, \quad y = 17.25 + 2.55(5) - .128(5^2) = \underline{26.8}$$
$$x=7, \quad y = 17.25 + 2.55(7) - .128(7^2) = \underline{28.83}$$




(19) --One may either extend the curve to see if trend is significant, or substitute future time-periods (forecast). That is, if $x=9$ is the present time-period, substitute $x=11$ and plot the point in the time-plot above:

$$\text{If } x=11, \; y = 17.25 + 2.55(11) - .128(121) = 29.8$$

If a standard and UCL=29.8 had been established, as shown on graph, the trend would be significant. However, if a <u>new</u> measurement, say at $x=10$, produced a more "level" curve (after computation and plotting), this curve could very well <u>not</u> touch the UCL at $x=11$ and the <u>new</u> curve (trend) would <u>not</u> be significant. Note that the value 30.2 is above 29.8. Obviously the trend was significant <u>at that point</u>, but not necessarily after <u>lower</u> measurements came in.

GUIDESHEET #7. The Single Moving Average

Assumptions:

1---This method is used where there is very little trend apparent from the time-plot, but one wishes to "smooth" random fluctuations in the past data. It is also a lead-in to more descriptive methods.

Steps:

1---Arrange the data in vertical columns (as in example) with column headings as follows. Column 4 will be filled in later.

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Single Moving Average $M_i$ |
|---|---|---|---|
| | | | |

2---Choose "n", the total number of observations or measurements which are to be tested for trend.

3---Choose "N", the number of previous points you wish to use in the computation of the Single Moving Average (SMA). If there are less than 15 total observations, use N=5, if there are more, one may use more than 5, but more than N=10 becomes unwieldy.

4---Compute the first N-period moving average. It is the simple average of the first N measurements. Use the following formula. Record this value in column 4.

$$M_i = \frac{\Sigma y_i}{N} = \frac{y_1+y_2+\cdots+y_N}{N} \qquad (1)$$

5---Once the first $M_i$ has been computed, an easier formula may be used for values up to the present. Record each in column 4.

$$M_i = M_{i-1} = \frac{y_i - y_{i-N}}{N} \tag{2}$$

6---Although they may not be needed, computing moving averages up to the last known value of y "smooths" the data. Plotting these points and connecting with short lines will show trends which appeared in the past.

7---Either superimpose the data and connecting lines onto a Control Chart (Guidesheet #3 or #4) or superimpose a Control Chart onto the graph completed in this Guidesheet. If preparing charts for a briefing, it may be convenient to use overlay transparencies.

8---The Control Chart limits (whether LCL or UCL or both) should be considered the point where the trend becomes significant. Future values are constant and equal to the latest moving average.

EXAMPLE for Single Moving Average (Guidesheet #7)

Steps:

1---

| Period Name (Flight Date) | Period Number $x_i$ | Measurement $y_i$ | N-Period Moving Average $M_i$ |
|---|---|---|---|
| 3 Dec | 1 | 18.7 | |
| 8 Jan | 2 | 14.9 | |
| 2 Feb | 3 | 6.07 | |
| 15 Mar | 4 | 5.73 | |
| 1 Apr | 5 | 9.35 | 10.95 |
| 28 May | 6 | 21.2 | 11.45 |
| 25 Jun | 7 | 20.6 | 12.59 |
| 19 Jul | 8 | 23.3 | 16.04 |
| 4 Aug | 9 | 23.3 | 19.55 |
| 8 Sep | 10 | 27.4 | 23.16 |
| 5 Oct | 11 | 30.2 | 24.96 |
| 18 Nov | 12 | 28.9 | 26.62 |
| 12 Dec | 13 | 29.6 | 27.88 |
| 12 Jan | 14 | 29.0 | 29.02 |

2---We wish to test all the data, so n=14

3---Since n=14<15, choose N=5.

4---Compute the first 5-period moving average:

$$M_5 = \frac{\sum_{i=1}^{5} y_i}{N} = \frac{y_1+y_2+y_3+y_4+y_5}{N}$$

$$= \frac{18.7+14.9+6.07+5.73+9.35}{5} = \frac{54.75}{5} = 10.95$$

$$M_5 = \underline{10.95}$$

5---Compute $M_6$

$$M_6 = M_{6-1} + \frac{y_6 - y_{6-5}}{5} = M_5 + \frac{y_6 - y_1}{5}$$

$$M_6 = 10.95 + \frac{21.2 - 18.7}{5} = 10.95 + \frac{2.5}{5}$$

$$= 10.95 + .5 = 11.45$$

$M_6$ = 11.45 Record $M_6$ in Column 4.

Compute moving averages up to $M_{14}$, using formula (1).

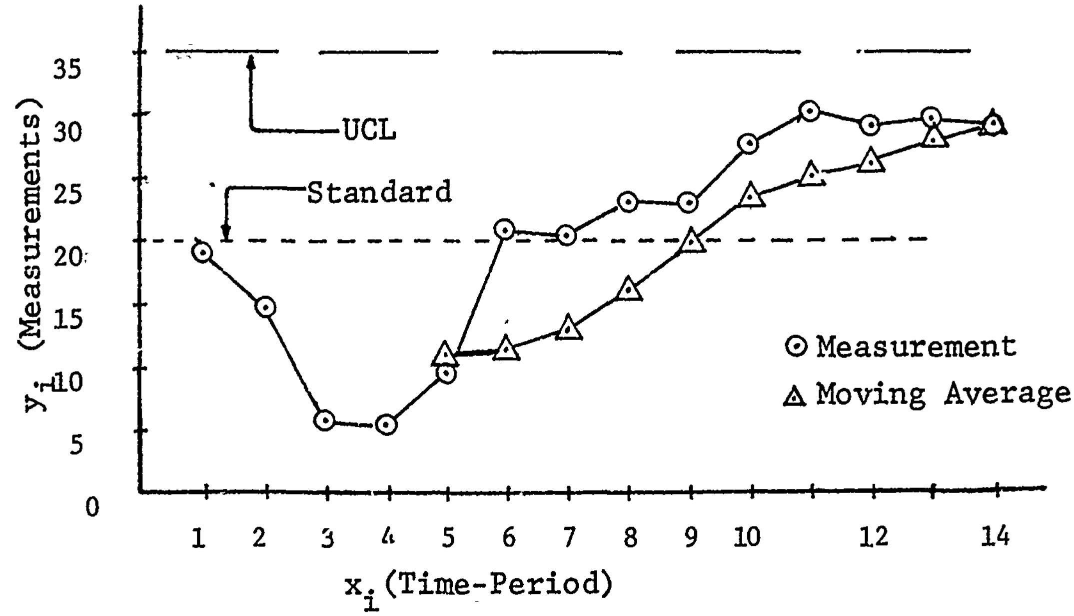
$$M_7 = M_{7-1} + \frac{y_7 - y_{7-5}}{5} = M_6 + \frac{y_7 - y_2}{5}$$

$$= 11.45 + \frac{20.6 - 14.9}{5} = 11.45 + 1.14 = 12.59$$

$M_7$ = 12.59 Record in Column 4.

$M_8$ through $M_{14}$ are computed and recorded as shown above.

6---Plot the data and all moving averages




All future (forecast) values are equal to $M_{14}$ until a new measurement comes in.

7---The time-plot above shows how the data may appear in conjunction with a standard and UCL (Guidesheet #3 or #4).

8---Since all forecast values for this method are equal to the latest moving average the above plot of moving averages "levels" off. Therefore, it would be hard to say this trend is significant unless a point had already touched the UCL.

GUIDESHEET #8. The Double Moving Average

Assumptions:

1---This method is to be used where the time-plot of the data suggests a straight-line trend, but one wishes not to have to compute a new straight-line equation each time a new measurement is taken.

Steps:

1---Arrange the data in vertical columne (as in example) with column headings as follows. Columns 4 and 5 will be filled in later.

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Single Moving Avg. $M_i^{[1]}$ | Double Moving Avg. $M_i^{[2]}$ |
|---|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

2---Choose "n" the total number of measurements that are to be tested for trend.

3---Choose "N", the number of previous points you with to use in the computation of the Single Moving Average and the Double Moving Average.

4---Compute the first N-period Single Moving Average according to the following formula.

$$M_i^{[1]} = \frac{\Sigma y_i}{N} = \frac{y_1+y_2+ \quad +y_N}{N} \qquad (1)$$

This first $M_i^{[1]}$ goes in the Nth space down the table, column #4.

5---Once the first $M_i^{[1]}$ has been computed, an easier formula may be used for subsequent values. These are then recorded in the table.

$$M_i^{[1]} = M_{i-1}^{[1]} + \frac{y_i - y_{i-N}}{N} \quad (2)$$

6---Compute the first N-period Double Moving Average $M_i^{\bullet}$ by averaging the first N Single Moving Averages according to the following formula. This goes in the N+Nth space, column #5.

$$M_i^{[2]} = \frac{\Sigma M_i^{[1]}}{N} = \frac{M_1^{[1]}+M_2^{[1]}+\cdots+M_N^{[1]}}{N} \quad (3)$$

7---Once the first Double Moving average has been computed an easier formula may be used for subsequent values. These are then recorded in the table.

$$M_i^{[2]} = M_{i-1}^{[2]} + \frac{M_i^{[1]} - M_{i-N}^{[1]}}{N} \quad (4)$$

8---Compute the quantity $a_i$, a quantity analogous to the y-intercept when finding the line-of-best-fit in Guidesheet #5.

$$a_i = 2M_i^{[1]} - M_i^{[2]} \quad (5)$$

9---Compute the quantity $b_i$, a quantity analogous to the slope in Guidesheet #5.

$$b_i = \left(\frac{2}{N-1}\right) \cdot (M_i^{[1]} - M_i^{[2]}) \quad (6)$$

10--Compute the forecast for T periods into the future using the following formula, which is much like the equation of the straight line developed in Guidesheet #5.

$$y_{i+T} = a_i + b_i \cdot T \quad (7)$$

Where i is the present period number and T is the number of periods ahead of i you wish to forecase.

Neither moving average need be plotted in this case, particularly if one forecast is all one is interested in. If this chart is to be maintained over time, however, the forecasts should be plotted as they are computed.

(11)--For trending, either superimpose the data and connecting lines onto a control chart (Guidesheet #3) or superimpose a control chart onto the graph completed in this guidesheet. If preparing charts for a briefing, it may be convenient to use overlay transparencies.

(12)--The control chart limits (LCL, UCL, or both) should be considered the point where the trend becomes significant. That is, near-term forecasts can be made and plotted. If one touches an established LCL or UCL, the trend is significant.

Notes:

(1)---When a new measurement is taken, a new forecast equation, Eq (7) must be computed.

EXAMPLE for Double Moving Average (Guidesheet #8)

Steps:

1---

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Single Moving Avg. $M_i^{[1]}$ | Double Moving Avg. $M_i^{[2]}$ |
|---|---|---|---|---|
| 3 Dec | 1 | 18.7 | | |
| 8 Jan | 2 | 14.9 | | |
| 2 Feb | 3 | 6.07 | | |
| 15 Mar | 4 | 5.73 | | |
| 1 Apr | 5 | 9.35 | 10.95 | |
| 28 May | 6 | 21.2 | 11.45 | |
| 25 Jun | 7 | 20.6 | 12.59 | |
| 19 Jul | 8 | 23.3 | 16.04 | |
| 4 Aug | 9 | 23.3 | 19.55 | 14.12 |
| 6 Sep | 10 | 27.4 | 23.16 | 16.56 |
| 5 Oct | 11 | 30.2 | 24.96 | 19.26 |
| 18 Nov | 12 | 28.9 | 26.62 | 22.07 |
| 12 Dec | 13 | 29.6 | 27.88 | 24.44 |
| 12 Jan | 14 | 29.0 | 29.02 | 26.33 |

2---We wish to use all the data for the test so n=14.

3---Choose N=5, the number of past data points used to predict future points.

4---Compute the first 5-period Single Moving Average (SMA).

$$M_5^{[1]} = \frac{\sum_{i=1}^{5} y_i}{5} = \frac{y_1+y_2+y_3+y_4+y_5}{5}$$

$$= \frac{18.7+14.9+6.07+5.73+9.35}{5} = \frac{54.75}{5} = 10.95$$

$$M_5^{[1]} = \underline{10.95}$$

5---Now compute the remaining 9 SMA's using formula (2)

$$M_i^{[1]} = M_{i-1}^{[1]} + \frac{y_i - y_{i-N}}{N} = M_{6-1}^{[1]} + \frac{y_6 - y_{6-5}}{5}$$

$$= M_5^{[1]} + \frac{y_6 - y_1}{5} = 10.95 + \frac{21.2 - 18.7}{5}$$

$$= 10.95 + \frac{2.5}{5} = 10.95 + .5 = 11.45$$

$$M_6^{[1]} = \underline{11.45}$$

Continue computations of SMA's and record above as shown.

(6)---Compute the first DMA, that is, the average of the first 5 SMA's according to formula (3).

$$M_i^{[2]} = \frac{\sum_{i=5}^{9} M_i^{[1]}}{5} = \frac{M_5^{[1]}+M_6^{[1]}+M_7^{[1]}+M_8^{[1]}+M_9^{[1]}}{5}$$

$$= \frac{10.95+11.45+12.59+16.04+19.55}{5}$$

$$= \frac{70.58}{5} = 14.12$$

$M_9^{[2]} = \underline{14.12}$ Record this value in Column #5,

as shown above.

(7)---Now compute the remaining 5 DMA's and record in Column #5. Once the first DMA is computed, Eq (4) may be used for subsequent values:

$$M_i^{[2]} = M_{i-1}^{[2]} + \frac{M_i^{[1]} - M_{i-N}^{[1]}}{N} = M_{10-1}^{[2]} + \frac{M_{10}^{[1]} - M_{10-5}^{[1]}}{5}$$

$$= M_9^{[2]} + \frac{M_{10}^{[1]} - M_5^{[1]}}{5} = 14.12 + \frac{23.16-10.95}{5}$$

$$= 14.12 + \frac{12.21}{5} = 14.12 + 2.44 = 16.56$$

$M^{[2]}_{10}$ = $\underline{16.56}$

Continue computations of DMA's, record, and check answers above.

8---Compute the quantity $a_i = 2M^{[1]}_i - M^{[2]}_i$

$$a_{14} = 2M^{[1]}_{14} - M^{[2]}_{14} = 2(29.02 - 26.33 =$$

$$= 58.04 - 26.33 = 31.71$$

$$a_{14} = \underline{31.71}$$

9---Compute the quantity $b_i = \frac{2}{N-1} (M^{[1]}_i - M^{[2]}_i)$

$$b_{14} = \frac{2}{5-1} (M^{[1]}_{14} - M^{[2]}_{14}) = \frac{2}{4}(29.02 - 26.33)$$

$$= \frac{1}{2}(2.69) = 1.34$$

$$b_{14} = \underline{1.34}$$

10--Any forecast T-periods in the future may be computed using Eq (7):

$$y_{i+T} = a_i + b_i T$$

If the present period is i=14, and we wish to forecast T=1 period into the future, we have:

$$y_{14+1} = a_{14}+b_{14}\cdot 1 = 31.71 + 1.34\cdot 1$$

$$= 31.71 + 1.34 = 33.05$$

$$y_{15} = \underline{33.05}$$

If we wish to forecast T=3 periods into the future we have:

$$y_{14+3} = a_{14} + b_{14}T = a_{14} + b_{14}\cdot 3$$

$$=31.71 + (1.34)(3) = 31.71 + 4.02 = 35.7$$

$y_{17}$ = $\underline{35.7}$

11 & 12---If there were a UCL established (Guidesheet #3 or #4) at y=35.0, this trend would be considered significant. When the new measurement $y_{15}$ comes in new values are used to write a new Eq (7). See time-plot below.

If $y_{15}$=27.0,

$$M^{[1]}_{15} = M^{[1]}_{14} + \frac{y_{15} - y_{10}}{5} = 29.02 + \frac{27.0 - 29.0}{5}$$

$$= 29.02 + \frac{-2}{5} = 29.02 - .4 = 28.62$$

$M^{[1]}_{15}$ = $\underline{28.62}$

Record in table and use in next computation.

$$M^{[2]}_{15} = M^{[2]}_{14} + \frac{M^{[1]}_{15} - M^{[1]}_{10}}{5} = 26.33 + \frac{28.62 - 23.16}{5}$$

$$= 26.33 + \frac{5.46}{5} = 26.33 + 1.09 = 27.42$$

$M^{[2]}_{15}$ = $\underline{27.42}$

Record in table and use in next computation.

$$a_{15} = 2M^{[1]}_{15} - M^{[2]}_{15} = 2(28.62) - 27.42$$

$$= 57.24 - 27.42 = 29.82$$

$a_{15}$ = $\underline{29.82}$

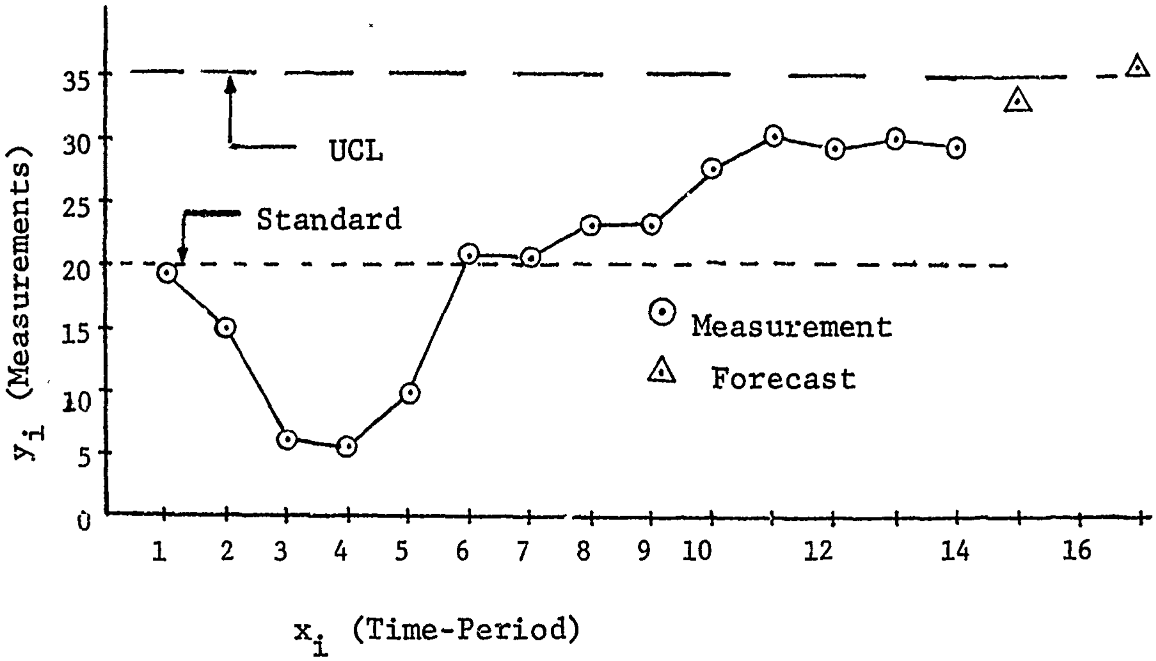
$$b_{15} = \left(\frac{2}{5-1}\right)(M^{[1]}_{15} - M^{[2]}_{15}) = \frac{1}{2}(28.62-27.42) = 0.6$$

$b_{15}$ = $\underline{0.6}$

and

$$y_{15+T} = 29.82 + (0.6)T$$

to forecast T periods into future.



GUIDESHEET #9. Triple Exponential Smoothing

Assumptions:

1---This method is to be used when a time-plot of the data suggests a curved-line trend, but one wishes not to have to compute a new curved-line equation each time a new measurement is taken

Steps:

1---Go directly to Guidesheet #6, Curvilinear Least-Squares-Best-Fit, and follow Step 2 through Step 16. At this point (Step 16) values will have been computed for unknowns called "a", "b", and "c". Return to this Guidesheet (Step 2) with those values.

2---Write down the values of a, b, and c computed in Guidesheet #6.

3---Arrange data in vertical columns (as in example) with column headings as follows. A new line for i=0 is added at the top for later use. Columns 1, 2, and 3 will be identical to those filled out for Step 1, Guidesheet #6. Columns 4, 5, and 6 will be filled in later.

| Period Name | Period Number $x_i$ | Measurement $y_i$ | $S_i^{[1]}$ | $S_i^{[2]}$ | $S_i^{[3]}$ |
|---|---|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

4---Choose the value of "r", the smoothing constant. It has been found that if an N of 5 gives good performance with the double moving average, an $r=\frac{2}{N+1}$ gives similar performance (Ref 8:92). This would give us r=.33. To make it easier to work with, a good starting value for our purposes is r=.3.

5---Compute the initial estimates for i=0 using the values of a, b, and c from Step 2 above. Use

equations (1), (2), and (3) below to compute $S_o^{[1]}$, $S_o^{[2]}$, and $S_o^{3}$.

$$S_o^{[1]} = a - \left(\frac{1-r}{r}\right)b + \left(\frac{(1-r)(2-r)}{r^2}\right)c \qquad (1)$$

$$S_o^{[2]} = a - \left(\frac{2(1-r)}{r}\right)b + \left(\frac{2(1-r)(3-2r)}{r^2}\right)c \qquad (2)$$

$$S_o^{[3]} = a - \left(\frac{3(1-r)}{r}\right)b + \left(\frac{3(1-r)(4-3r)}{r^2}\right)c \qquad (3)$$

6 ---For r=.3, the equations can be written as follows.

$$S_o^{[1]} = a - 2.33b + 13.2c \qquad (4)$$

$$S_o^{[2]} = a - 4.67b + 37.3c \qquad (5)$$

$$S_o^{[3]} = a - 7b + 72.3c \qquad (6)$$

For other equations for different values of "r", see Note #1.

7 ---Compute the single, double, and triple exponentially smoothed statistics for time i=1 using the following formulas. Record these values in Columns 4, 5, and 6.

$$S_1^{[1]} = .3y_1 + .7S_o^{[1]} \qquad (7)$$

$$S_1^{[2]} = .3S_1^{[1]} + .7S_o^{[2]} \qquad (8)$$

$$S_1^{[3]} = .3S_1^{[2]} + .7S_o^{[3]} \qquad (9)$$

8 ---Continue computing the exponentially smoothed statistics for i=2 through the present time-period using formulas (10), (11), and (12) shown here. Record each statistic in the appropriate columns.

$$S_i^{[1]} = .3y_i + .7S_{i-1}^{[1]} \tag{10}$$

$$S_i^{[2]} = .3S_i^{[1]} + .7S_{i-1}^{[2]} \tag{11}$$

$$S_i^{[3]} = .3S_i^{[2]} + .7S_{i-1}^{[3]} \tag{12}$$

(9)---Compute the coefficients of the triple exponentially smoothed model $a_i$, $b_i$, and $c_i$, according to the following formulas.

$$a_i = 3S_i^{[1]} - 3S_i^{[2]} + S_i^{[3]} \tag{13}$$

$$b_i = .31(4.5S_i^{[1]} - 7.6S_i^{[2]} + 3.1S_i^{[3]}) \tag{14}$$

$$c_i = .092(S_i^{[1]} - 2S_i^{[2]} + S_i^{[3]}) \tag{15}$$

(10)--Write the forecasting equation for period "i" with coefficients from Step 9 as follows.

$$y_{i+T} = a_i + b_i \cdot T + c_i \cdot T^2 \tag{16}$$

Where T denotes the number of periods in the future one wishes to forecast. That is, the forecast for T=1 period in the future would be:

$$y_{i+T} = y_{i+1} = a_i + b_i + c_i \tag{17}$$

(11)--Whenever a new data-point must be computed, new statistics $S_i^{[1]}$, $S_i^{[2]}$, and $S_i^{[3]}$ must be computed. These are computed by formulas (10), (11), and (12) from Step 8. These are recorded in the table and used in Step 12.

(12)--Using the new statistics from Step 11 and formulas (13), (14), and (15) from Step 9. we compute new coefficients $a_i$, $b_i$, and $c_i$.

(13)--With the new coefficients $a_i$, $b_i$, and $c_i$ computed in Step 12, we may write the new forecasting formula.

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$$y_{i+T} = a_i + b_i \cdot T + c_i \cdot T^2 \qquad (18)$$

Again, T is the number of periods in the future a forecast is needed.

(14) --When used in conjunction with a control chart, one may say a trend is significant at the $\alpha$ level of significance (see Guidesheet #3 or #4) when a forecast touches the appropriate control chart limit.

Notes:

(1)---For a smoothing constant of r=.1 instead of r=.3 our equations (4), (5), (6), (10) through (15) are as follows:

$$S_o^{[1]} = a - 9b + 17c \qquad (4)$$

$$S_o^{[2]} = a - 18b + 504c \qquad (5)$$

$$S_o^{[3]} = a - 27b + 999c \qquad (6)$$

$$S_i^{[1]} = .1y_i + .9S_{i-1}^{[1]} \qquad (10)$$

$$S_i^{[2]} = .1S_i^{[1]} + .9S_{i-1}^{[2]} \qquad (11)$$

$$S_i^{[3]} = .1S_i^{[2]} + .9S_{i-1}^{[3]} \qquad (12)$$

$$a_i = 3S_i^{[1]} - 3S_i^{[2]} + S_i^{[3]} \text{ (no change)} \qquad (13)$$

$$b_i = .062(5.5S_i^{[1]} - 9.2S_i^{[2]} + 3.7S_i^{[3]}) \qquad (14)$$

$$c_i = .0062(S_i^{[1]} - 2S_i^{[2]} + S_i^{[3]}) \qquad (15)$$

If it is necessary to use other values of r, the main text of the thesis must be consulted for the general formulas.

EXAMPLE for Triple Exponential Smoothing (Guidesheet #9)

Steps:

(1)---We will use the same data as the example for Guidesheet #6.

(2)---a=17.25, b=2.55, c=-.128

(3)---Table as shown:

| Period Name Half-Yr | Period Number $x_i$ | Measurement $y_i$ | $S_i^{[1]}$ | $S_i^{[2]}$ | $S_i^{[3]}$ |
|---|---|---|---|---|---|
| | 0 | | 9.62 | 0.567 | -9.85 |
| 1st '74 | 1 | 21.2 | 13.09 | 4.33 | -5.60 |
| 2nd '74 | 2 | 20.6 | 15.34 | 7.63 | -1.63 |
| 1st '75 | 3 | 23.3 | 17.73 | 10.66 | 2.05 |
| 2nd '75 | 4 | 23.3 | 19.40 | 13.28 | 5.42 |
| 1st '76 | 5 | 27.4 | 21.80 | 15.84 | 8.55 |
| 2nd '76 | 6 | 30.2 | 24.32 | 18.38 | 11.50 |
| 1st '77 | 7 | 28.9 | 25.69 | 20.57 | 14.22 |
| 2nd '77 | 8 | 29.6 | 26.86 | 22.46 | 16.69 |
| 1st '78 | 9 | 29.0 | 27.50 | 23.97 | 18.87 |

(4)---We choose an "r": r=0.3

(5)---Skip to the worked-out formulas from Step 6.

(6)---Compute the estimates for the <u>initial</u> exponentially smoothed statistics according to equations (4), (5) and (6).

$$S_0^{[1]} = a-2.33b+13.2c = 17.25-2.33(2.55)+13.2(-.128)$$

$$= 17.25 -5.94 - 1.69 = 9.62$$

$$S_0^{[1]} = \underline{9.62}$$

Record this value above, across from $x_i$=0 in Column #4.

$$S_0^{[2]} = a-4.67b+37.3c = 17.25-4.67(2.55)+37.3(-.128)$$

$$= 17.25 - 11.91 - 4.77 = 0.567$$

$S_o^{[2]} = \underline{0.567}$ Record this value above.

$S_o^{[3]} = a-7b+72.3c = 17.25-7(2.55)+72.3(-.128)$

$= 17.25 - 17.85 - 9.25 = -9.85$

$S_o^{[3]} = \underline{-9.85}$ Record above.

7 ---Compute the single, double, and triple exponentially smoothed statistics using (7), (8), and (9):

$S_1^{[1]} = .3y_i+.7S_o^{[1]} = .3(21.2)+.7(9.62)$

$= 6.36 + 6.73 = \underline{13.09}$

Record in table above as shown.

$S_1^{[2]} = .3S_1^{[1]} + .7S_o^{[2]} = .3(13.09)+.7(0.57)$

$= 3.93 + .40 = \underline{4.33}$ Record above as shown.

$S_1^{[3]} = .3S_1^{[2]} + .7S_o^{[3]} = .3(4.33)+.7(-9.85)$

$= 1.30 - 6.90 = -\underline{5.60}$ Record above.

8 ---We use the generalized formulas (10), (11), and (12) to compute the remaining 8 values in Columns 4, 5, and 6 of the table as follows:

$S_i^{[1]}$

For i=2,

$S_i^{[1]} = .3y_i+.7S_{i-1}^{[1]}$ (Eq 10)

$=.3y_2+.7S_{2-1}^{[1]} = .3(20.6)+.7S_1^{[1]}$

$= 6.18+.7(13.09) = 6.18+9.16 = 15.34$

$S_2^{[1]} = \underline{15.34}$

For i=3,

$S_3^{[1]} = .3y_3+.7S_{3-1}^{[1]} = .3(23.3)+.7(S_2^{[1]}) = 6.99+.7(15.34)$

$= 6.99 + 10.74 = \underline{17.73}$

and so on, getting the values shown in Column #4, above.

$S_i^{[2]}$ {

For i=2,

$$S_i^{[2]} = .3S_i^{[1]} + .7S_{i-1}^{[2]} \qquad \text{(Eq 11)}$$

$$S_2^{[2]} = .3S_2^{[1]}+.7S_{2-1}^{[2]} = .3(15.34)+.7(S_1^{[2]})= 4.60+.7(4.33)$$

$$=4.60 + 3.03 = \underline{7.63}$$

For i=3,

$$S_3^{[2]} = .3S_3^{[1]}+.7S_2^{[2]} = .3(17.73)+.7(7.63) = \underline{10.66}$$

and so on, getting the values shown in Column #5, above.

$S_i^{[3]}$ {

For i=2,

$$S_i^{[3]} = .3S_i^{[2]} + .7S_{i-1}^{[3]} \qquad \text{(Eq 12)}$$

$$S_2^{[3]} = .3S_2^{[2]} + .7S_1^{[3]} = .3(7.63)+.7(-5.60) = \underline{-1.63}$$

For i=3,

$$S_3^{[3]} = .3S_3^{[2]}+.7S_2^{[3]} = .3(10.66)+.7(-1.63) = \underline{2.05}$$

and so on, getting the values shown in Column #6.

9 ---We now compute the coefficients $a_i$, $b_i$, and $c_i$ using formulas (13), (14), and (15). Since we are in period 9, i=9.

$$a_9 = 3S_9^{[1]}-3S_9^{[2]}+S_9^{[3]} = 3(27.50)-3(23.97)+18.87$$

$$= 82.50-71.91+18.87 = \underline{29.46}$$

$$b_9 = .31\left(4.5S_9^{[1]} - 7.6S_9^{[2]} + 3.1S_9^{[3]}\right)$$

$$=.31\left(4.5(27.50)-7.6(23.97)+3.1(18.87)\right)$$

$$=.31(.075) = \underline{.023}$$

$$c_9 = .092\left(S_9^{[1]}-2S_9^{[2]}+S_9^{[3]}\right) = .092\left(27.50-2(23.97)+18.87\right)$$

$$=.092(-1.57) = \underline{-.144}$$

(10)--We may now write our forecasting equation as Eq (16) shows:

$$y_{i+T} = a_i + b_i \cdot T + c_i \cdot T^2$$

or, at the present time (i=9) we have:

$$y_{9+T} = a_9 + b_9 \cdot T + c_9 \cdot T^2$$

If we wish to forecast 1 period beyond i=9, T=1, $a_i$, $b_i$, and $c_i$ are from Step 9 and we have:

$$y_{9+1} = 29.46 + .023(1) - .144(1^2)$$

$$y_{10} = 29.46 + .023 - .144 = 29.339 \text{ or } \underline{29.3}$$

If we wished to forecast 3 periods, we would have:

$$y_{9+3} = y_{12} = 29.46+.023(3)+.144(3^2)$$

$$= 29.46+.069-.144(9) = \underline{28.2}$$

(11)--Say the next period's measurement comes in, i.e. $y_{10}$=27.0. We must first compute the statistics $S^{[1]}_{10}$ $S^{[2]}_{10}$, and $S^{[3]}_{10}$. We again use equations (10), (11), and (12):

$$S^{[1]}_{10} = .3y_{10}+.7S^{[1]}_{9} = .3(27.0)+.7(27.5) = \underline{27.35}$$

$$S^{[2]}_{10} = .3S^{[1]}_{10}+.7S^{[2]}_{9} = .3(27.35)+.7(23.97) = 24.98$$

$$S^{[3]}_{10} = .3S^{[2]}_{10}+.7S^{[3]}_{9} = .3(24.98)+.7(18.87) = 20.70$$

(12)--We may now compute the coefficients for our new forecasting equation using equations (13), (14), and (15):

$$a_{10} = 3S^{[1]}_{10}-3S^{[2]}_{10}+S^{[3]}_{10} = 3(27.35)-3(24.98)+20.7$$

$$= \underline{27.8}$$

$$b_{10} = .31(4.5S^{[1]}_{10} - 7.6S^{[2]}_{10} + 3.1S^{[3]}_{10})$$

$$= .31(4.5(27.35) - 7.6(24.98) + 3.1(20.7))$$

$$= .31(-2.60) = \underline{-.807}$$

$$c_{10} = .092(S^{[1]}_{10} - 2S^{[2]}_{10} + S^{[3]}_{10})$$

$$= .092(27.35 - 2(24.98) + 20.7) = .092(-1.91)$$

$$= \underline{-.176}$$

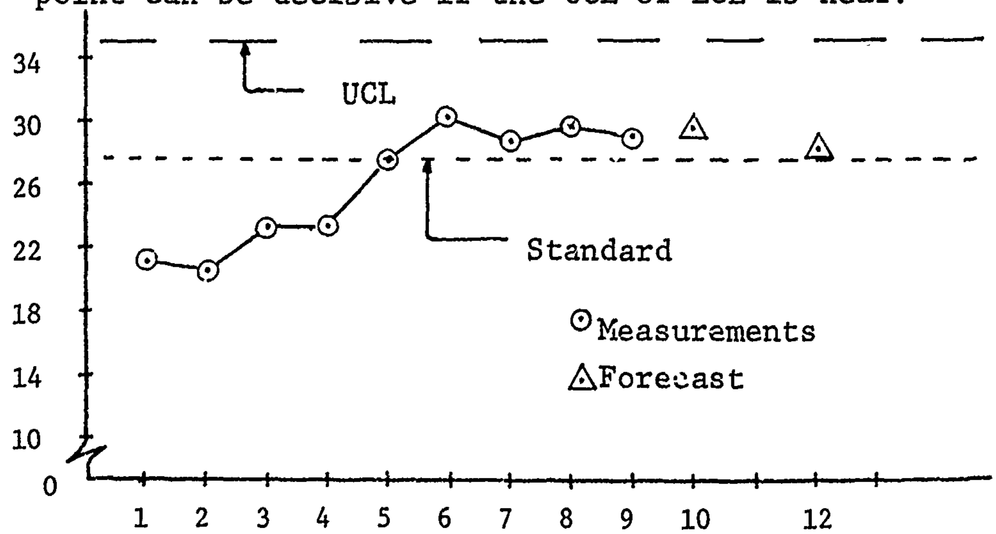
13 --With these new values of a, b, and c we may write a new forecasting equation:

$$y_{10+T} = a_{10} + b_{10} \cdot T + c_{10} \cdot T^2$$

$$y_{10+T} = 27.8 - .807 \cdot T - .176 \cdot T^2$$

14 --The data is plotted as follows. If there were a standard as shown and a UCL at y=35.0, a trend would be significant (at chosen $\alpha$) if a near-term forecast reached the UCL. In our example it does not.

Note that the forecast for $y_{12}$ in Step 10 is 28.2. The forecast for $y_{12}$ using the new equation in Step 13 is $y_{12}$=25.5. This shows the effect of a new data point can be decisive if the UCL or LCL is near.



GUIDESHEET #10. Parametric Test for Correlation

Assumptions:

1---It is assumed that there are 2 types of measurements here, and that one is interested in whether there is positive, negative, or zero relationship between them. This is not a trend test.

Steps:

1---Arrange the data in vertical columns (as in example) with column headings as follows. Columns 3 through 6 will be filled in later.

| Measurement #1 $x_i$ | Measurement #2 $y_i$ | Square $x_i^2$ | Square $y_i^2$ | Product $x_i \cdot y_i$ |
|---|---|---|---|---|
| | | | | |
| $\Sigma x_i =$ ___ | $\Sigma y_i =$ ___ | $\Sigma x_i^2 =$ ___ | $\Sigma y_i^2 =$ ___ | $\Sigma x_i y_i =$ ___ |

2---Plot the measurements jointly, one axis for each measurement. Do not connect any of the plotted points with lines.

3---If it will simplify computation, the measurements may be divided by 10, 100, or whatever. If there are more than three significant digits, also, round off to three. These two things will not affect the test significantly and will prevent working with extremely large numbers. During computation, however, carry as many significant figures as possible Likewise, if the numbers are very small (.000236 for example) and vary in the fourth to sixth decimal place, say, multiply by a large number (1000 in this case) to make the numbers manageable.

4---Compute the square of each $x_i$ measurement. That is, compute $x_i^2 = x_i \cdot x_i$ and record in the designated column.

5---Compute the square of each $y_i$ measurement. That is, compute $y_i^2=y_i \cdot y_i$ and record in the designated column.

6---Compute the product of each $x_i$ times each $y_i$. That is, compute the product $x_i \cdot y_i$, and record in the last column.

7---Add up each column to compute the following sums: $\Sigma x_i$, $\Sigma y_i$, $\Sigma x_i^2$, $\Sigma y_i^2$, $\Sigma x_i \cdot y_i$. Record each total in the designated place at the bottom of each column.

8---Compute the quantity of $S_{xx}$ according to the following formula, where "n" is the number of measurements.

$$S_{xx} = n\sum x_i^2 - (\sum x_i)^2 \tag{1}$$

9---Compute the quantities $S_{yy}$ and $S_{xy}$ according to the following formulas.

$$S_{yy} = n\sum y_i^2 - (\sum y_i)^2 \tag{2}$$

$$S_{xy} = n\sum x_i \cdot y_i - (\sum x_i) \cdot (\sum y_i) \tag{3}$$

10--Compute the Sample Correlation Coefficient "r", according to the following formula.

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} \tag{4}$$

11--Choose a significance level $\alpha$.

12--Look up the value of the z-statistic corresponding to the $\alpha/2$ picked in Step 11 from the table in Note 3. This is the table value of the test statistic z(Table).

13--Compute the value of Z, an intermediate statistic, according to the following formula.

$$Z = \tfrac{1}{2}\ln\left(\frac{1+r}{1-r}\right) \tag{5}$$

If calculator does not have the ln function, the table in Note 4 gives values of Z for various values of r.

(14)--Compute the computed value of the test statistic, z(Computed) according to the following formula, using Z from Step 13, and n from Step 8.

$$z(\text{Computed}) = \sqrt{n-3} \cdot Z$$

(15)--If z(Computed) < z(Table) for positive values of r, then there is no significant correlation. If z(Computed) > -z(Table) for negative values of r, there is no significant negative correlation. If z(Computed) > z(Table) for positive values of 4, there <u>is</u> a positive correlation, and if z(Computed) < -z(Table) for negative values of r, there <u>is</u> significant negative correlation.

Notes:

(1)---Step 13 may be regarded as a test of the hypothesis $H_o: \rho=0$ where $\rho$ is the value of the true but unknown relationship between the measurements. The alternative hypothesis is $H_1: \rho \neq 0$. If z(Computed) lies between -z(Table) and z(Table) we accept $H_o$. If z(Computed) lies outside these limits, we must reject $H_o$.

(2)---Any <u>significant</u> correlation may be due to a third factor, or in rare cases ($\alpha$% of the time), chance. Take this into consideration when making decisions about what causes what.

(3)---Table values of the z-statistic are given here:

| | | | |
|---|---|---|---|
| $\alpha$ = | 0.10 | 0.05 | 0.01 |
| $\alpha/2$ = | 0.05 | 0.025 | 0.005 |
| z(Table) = | 1.645 | 1.96 | 2.575 |

④---The intermediate statistic Z for various values of "r" are given below:

VALUES OF $Z = \frac{1}{2} \ln \frac{1+r}{1-r}$

| $r$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.000 | 0.010 | 0.020 | 0.030 | 0.040 | 0.050 | 0.060 | 0.070 | 0.080 | 0.090 |
| 0.1 | 0.100 | 0.110 | 0.121 | 0.131 | 0.141 | 0.151 | 0.161 | 0.172 | 0.182 | 0.192 |
| 0.2 | 0.203 | 0.213 | 0.224 | 0.234 | 0.245 | 0.255 | 0.266 | 0.277 | 0.288 | 0.299 |
| 0.3 | 0.310 | 0.321 | 0.332 | 0.343 | 0.354 | 0.365 | 0.377 | 0.388 | 0.400 | 0.412 |
| 0.4 | 0.424 | 0.436 | 0.448 | 0.460 | 0.472 | 0.485 | 0.497 | 0.510 | 0.523 | 0.536 |
| 0.5 | 0.549 | 0.563 | 0.576 | 0.590 | 0.604 | 0.618 | 0.633 | 0.648 | 0.662 | 0.678 |
| 0.6 | 0.693 | 0.709 | 0.725 | 0.741 | 0.758 | 0.775 | 0.793 | 0.811 | 0.829 | 0.848 |
| 0.7 | 0.867 | 0.887 | 0.908 | 0.929 | 0.950 | 0.973 | 0.996 | 1.020 | 1.045 | 1.071 |
| 0.8 | 1.099 | 1.127 | 1.157 | 1.188 | 1.221 | 1.256 | 1.293 | 1.333 | 1.376 | 1.422 |
| 0.9 | 1.472 | 1.528 | 1.589 | 1.658 | 1.738 | 1.832 | 1.946 | 2.092 | 2.298 | 2.647 |

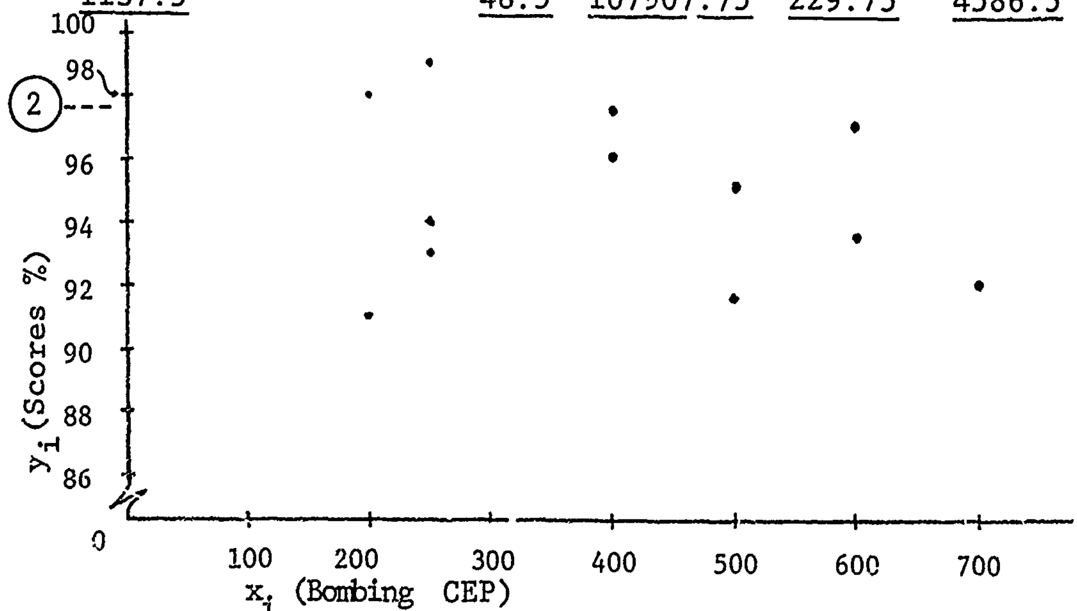
For negative values of $r$ put a minus sign in front of the corresponding $Z$'s, and vice versa.

EXAMPLE for Parametric Test for Correlation (Guidesheet #10)

Steps:

1---This is an example of how a trend analysis officer might test the relationship between proficiency test scores and bombing CEPs. The scores and CEPs are arranged as follows. Columns 3-6 are filled in later. This is <u>not</u> a test for trend.

| Measurement #1 (Test Score) $x_i$ | Measurement #2 (CEP) $y_i'$ | $y_i = \frac{y_i'}{100}$ | Square $x_i^2$ | Square $y_i^2$ | Product $x_i \cdot y_i$ |
|---|---|---|---|---|---|
| 91 | 200 | 2 | 8281 | 4 | 182 |
| 91.5 | 500 | 5 | 8372.25 | 25 | 457.5 |
| 92 | 700 | 7 | 8464 | 49 | 644 |
| 93 | 250 | 2.5 | 8649 | 6.25 | 232.5 |
| 93.5 | 600 | 6 | 8742.25 | 36 | 561 |
| 94 | 250 | 2.5 | 8836 | 6.25 | 235 |
| 95 | 500 | 5 | 9025 | 25 | 475 |
| 96 | 400 | 4 | 9216 | 16 | 384 |
| 97 | 600 | 6 | 9409 | 36 | 582 |
| 97.5 | 400 | 4 | 9506.25 | 16 | 390 |
| 98 | 200 | 2 | 9604 | 4 | 196 |
| 99 | 250 | 2.5 | 9801 | 6.25 | 247.5 |
| $\Sigma x_i =$ <u>1137.5</u> | | $\Sigma y_i =$ <u>48.5</u> | $\Sigma x_i^2 =$ <u>107907.75</u> | $\Sigma y_i^2 =$ <u>229.75</u> | $\Sigma x_i \cdot y_i =$ <u>4586.5</u> |



(3)---In order to avoid very large numbers, measurements $y_i'$ are divided by 100 become our $y_i$ values, and are recorded in Column #3.

(4)---Computing each square of Test Scores we have:

$x_i^2 = x_i \cdot x_i$ or

$x_1^2 = x_1 \cdot x_1 = 91 \cdot 91 = 8281$ Record in Column #4

$x_2^2 = x_2 \cdot x_2 = 91.5 \cdot 91.5 = 8372.25$ " " " "

$\vdots \quad \vdots \quad \vdots \quad \vdots$ " " " "

$x_{12}^2 = x_{12} \cdot x_{12} = 99 \cdot 99 = 9801$ " " " "

(5)---Computing each square of Bombing CEPs we have:

$y_i^2 = y_i \cdot y_i$ or

$y_1^2 = y_1 \cdot y_1 = 2 \cdot 2 = 4$ Record in Column #5.

$y_2^2 = y_2 \cdot y_2 = 5 \cdot 5 = 25$ " " " "

$\vdots \quad \vdots \quad \vdots \quad \vdots$ " " " "

$y_{12}^2 = y_{12} \cdot y_{12} = 2.5 \cdot 2.5 = 6.25$ " " " "

(6)---Computing each product of measurements we have:

$x_i \cdot y_i$ or

$x_1 \cdot y_1 = 91 \cdot 2 = 182$ Record in Column #6

$x_2 \cdot y_2 = 91.5 \cdot 5 = 457.5$ " " " "

$\vdots \quad \vdots \quad \vdots$ " " " "

$x_{12} \cdot y_{12} = 99 \cdot 2.5 = 247.5$ " " " "

(7)---Computing the sum of each column we have:

$\Sigma x_i = x_1 + x_2 + \cdots + x_{12} = 91 + 91.5 + \cdots + 99 = \underline{1137.5}$

Record as shown.

$$\Sigma y_i = y_1+y_2+\cdots+y_{12} = 2+5+\cdots+2.5 = \underline{48.5}$$

Record as shown.

$$\Sigma x_i^2 = x_1^2+x_2^2+\cdots+x_{12}^2 = 8281+8372.25+\cdots+9801=\underline{107905.75}$$

Record as shown.

$$\Sigma y_i^2 = y_1^2+y_2^2+\cdots+y_{12}^2 = 4+25+\cdots+6.25 = \underline{229.75}$$

Record as shown.

$$\Sigma x_i \cdot y_i = x_1 \cdot y_1+x_2 \cdot y_2+\cdots+x_{12} \cdot y_{12}$$

$$= 182+457.5+\cdots+247.5 = \underline{4586.5}$$

Record as shown.

8---There are 12 measurements of each kind so n=12. Computing the quantity $S_{xx}$ according to Eq (1), we have:

$$S_{xx} = n\sum x_i^2 - (\sum x_i)^2$$

$$=12(107905.75)-(1137.5)^2$$

$$=1,294,869 - 1,293,906.25$$

$$S_{xx} = \underline{962.75}$$

$S_{xx}$ will be used in Step 10.

9---Computing the quantities $S_{yy}$ and $S_{xy}$ according to Eqs (2) and (3) we have:

$$S_{yy} = n\sum y_i^2 - (\sum y_i)^2$$

$$=12(229.75)-(48.5)^2 = 2757-2352.25$$

$$S_{yy} = \underline{404.75}$$

$$S_{xy} = n\sum x_i \cdot y_i - (\sum x_i)(\sum y_i)$$

$$= 12(4586.5)-(1137.5(48.5)=55,038-55,168.75$$

$$S_{xy} = \underline{-130.75}$$

(10) --Computing the Sample Correlation Coefficient "r" according to Eq (4) we have:

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}}$$

$$= \frac{-130.75}{\sqrt{(962.75)(404.75)}} = \frac{-130.75}{\sqrt{389673.0625}}$$

$$r = \frac{-130.75}{624.2379855} = -.209455 \quad \text{or} \quad \underline{-.21}$$

(11) --We choose $\alpha=.05$. Other common values of $\alpha$ are $\alpha=.01$ and $\alpha=.10$.

(12) --Referring to the table in Note 3 we have z(Table) =1.96.

(13) --Entering the table in Note 4 with r=-.21 we have Z=-.213

(14) --Our z(Computed) is given by Eq (6) as follows:

$$z(\text{Computed}) = \sqrt{n-3} \cdot Z$$

$$z(\text{Computed}) = \sqrt{12-3}(-.213) = \sqrt{9}(-.213) = 3(-.213)$$

$$z(\text{Computed}) = \underline{-.639}$$

(15) --Since "r" is negative, r=-.21 we ask if z(Computed) is greater than the <u>negative</u> of z(Table). Or, put another way:

$$z(\text{Computed}) \overset{?}{\geq} -z(\text{Table})$$
$$-.639 \overset{?}{\geq} -1.96$$

Yes, it is greater. We conclude that there is no significant correlation at this $\alpha = .05$ level of significance. That is, on the basis of this sample, we cannot say that high proficiency scores are associated with low CEPs.

Here is an example of Note 2, page 109. In our example we should not say high scores cause low CEPs. It may be more appropriate to say that high proficiency (third factor) causes both high proficiency scores and low CEPs. But on the other hand the third factor could be cheating, which also causes high scores and low CEPs. The real thing going on is hard to measure. Statistics just helps measure the strength of a relationship.

GUIDESHEET #11. The Cox and Stuart Test for Trend

Assumptions:

1---This method is used when the assumptions required for Control-Chart significance-testing are not valid assumptions. Also, this method allows quicker significance-testing in most cases, though it is not as powerful a test.

Steps:

1---Arrange the data in vertical columns (as in example) with column headings as follows.

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Tally |
|---|---|---|---|
| | | | ~~||||~~ || |

2---Count the total number of observations or measurements that are to be tested for trend. Call this number "N".

3---If N is odd, discard the middle value by drawing a heavy line through the middle period, dividing the data into two equal parts. If N is even, draw the heavy line between the first and second halves, dividing the data into equal parts. See example.

4---Draw a line connecting the 1st measurement ($y_1$) in the 1st half to the 1st measurement in the 2nd half. As this is done, note whether the 2nd half value is greater. If it is, mark a tally in the "tally" column to the right of the table.

5---Continue connecting corresponding measurements and continue tallying each time the second half value is greater than the first half value.

6---Count the total number of lines. This is called "n". It should be half the number of measurements after discarding.

7---Count the tally. This is T(Computed)

8---Choose a significance level $\alpha$. A significance level of $\alpha$=.05 or $\alpha$=.10 is usually appropriate.

9---Form the null hypothesis $H_o$ and alternative hypothesis $H_1$. These may be picked from the three choices below.

(A) $H_o$: There is no downward trend.

$H_1$: There is a downward trend.

(B) $H_o$: There is no upward trend.

$H_1$: There is a significant upward trend in the data

(C) $H_o$: There is no significant trend.

$H_1$: There is either an upward trend or a downward trend in the data.

If (A) is chosen, continue with Step 10. If (B) is chosen, go to Step 15. If (C) is chosen, go to Step 20.

10--Enter Table II on page 123, look up and down the column of numbers associated with p=.50 and the n from Step 6. Find the 4-place decimal most nearly equal to the chosen value of $\alpha$ (Step 8).

11--Pick the value directly to the left of the 4-place decimal found in Step 10 (in the column labeled "x") and call this value T(Table).

12--If T(Computed) from Step 7 is greater than or equal to T(Table), accept the null hypothesis $H_o$: There is no downward trend. That is, if T(Computed)$\geq$T(Table), accept $H_o$.

(13)--If T(Computed) is less than T(Table), reject the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is a significant downward trend. That is, if T(Computed)<T(Table), accept $H_1$.

(14)--The 4-place decimal found in Step 10 is the <u>actual</u> significance level for the value of the computed test-statistic for this data. This step completes the test for a downward trend.

---

(15)--Enter Table II on page 123, look up and down the column of numbers associated with p=.50 and the n from Step 6. Find the 4-place decimal most nearly equal (preferably less than) the chosen value of α (Step 8).

(16)--Pick the value directly to the left of the 4-place decimal found in Step 15 (in the column labeled "x"), and subtract this number from n. This is the table value of the test-statistic T(Table).

(17)--If T(Computed) from Step 7 is less than or equal to T(Table), accept the null hypothesis $H_o$: There is no upward trend. That is, if T(Computed)≤T(Table), accept $H_o$.

(18)--If T(Computed) is greater than T(Table), reject the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is a significant upward trend. That is, if T(Computed)>T(Table), accept $H_1$.

(19)--The 4-place decimal found in Step 15 is the <u>actual</u> significance level for the value of the computed test-statistic and this data. This step completes the test for an upward trend.

---

(20)--Enter Table II and look up and down the column of numbers associated with p=.50 and the n from Step 6.

Find the 4-place decimal most nearly equal (preferably less than) to $\alpha/2$ where $\alpha$ is the value chosen in Step 8.

(21) --Pick the value directly to the left of the 4-place decimal found in Step 20 (in the column labeled "x"), and call this value T(Table), the table value of the test-statistic.

(22) --If T(Computed) from Step 7 is greater than or equal to T(Table) <u>and</u> less than or equal to n minus T(Table), accept the null hypothesis $H_o$: There is no significant trend. That is, if $T(\text{Table}) \leq T(\text{Computed}) \leq n - T(\text{Table})$, accept the null hypothesis $H_o$.

(23) --If T(Computed is less than T(Table) <u>or</u> greater than n minus T(Table), reject the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is either an upward or downward trend. That is, if $T(\text{Computed}) < T(\text{Table})$ <u>or</u> $T(\text{Computed}) > n - T(\text{Table})$, reject $H_o$ in favor of $H_1$.

(24) --The 4-place decimal (multiplied by 2) found in Step 20 is the <u>actual</u> significance level for this data and this test-statistic. This step completes the test for trend.

EXAMPLE for Cox and Stuart Test for Trend (Guidesheet #11)

Steps:

1---

| Period Name | Period Number $x_i$ | Measurement $y_i$ |
|---|---|---|
| Jan | 1 | 89.9 |
| Feb | 2 | 90.5 |
| Mar | 3 | 92.2 |
| Apr | 4 | 91.2 |
| May | 5 | 90.7 |
| Jun | 6 | 91.3 |
| Jul | 7 | 90.7 |
| ~~Aug~~ | ~~8~~ | ~~91.7~~ |
| Sept | 9 | 89.9 |
| Nov | 10 | 90.1 |
| Dec | 11 | 90.8 |
| Jan | 12 | 90.5 |
| Feb | 13 | 91.4 |
| Mar | 14 | 89.9 |
| Apr | 15 | 91.1 |

TALLY

T(Computed)= //

2---We wish to use all available data for this test, because we feel it is all relevant. Therefore N=15.

3---Since N is odd, we discard the middle value, $y_8$. Draw a heavy line through period #8 to divide the measurements into a first half and a second half, as shown above.

4---Draw a line connecting the 1st measurement in the first half to the 1st measurement in the second half. We note 2nd half value is <u>not</u> greater, but the same, so we do not tally.

5---As we continue connecting measurements, we find only one value in the second half is greater than the corresponding value in the first half.

6---n=7

7---The number in the tally is 2. Therefore T(Computed) =2, our computed test-statistic.

(8)---We choose a significance level $\alpha$=.05 for this test.

(9)---We are interested in whether there is a significant downward trend or not and do not care to tell if there is an upward trend, so we choose (A).

(10)--We go to Table II, starting on page 123, and look at the column associated with p=.50 and n=7. We look for the 4-place decimal closest to $\alpha$=.05. We find .0625 is the closest in the column. A portion of the table is shown with these values marked. The .0625 now becomes the exact significance level for this test. That is, $\alpha$=.0625.

| | | *p* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *n* | *x* | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| 2 | 2 | 0.9025 | 0.8100 | 0.7225 | 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 |
| | 1 | 0.9975 | 0.9900 | 0.9775 | 0.9600 | 0.9375 | 0.9100 | 0.8755 | 0.8400 | 0.7975 | 0.7500 |
| 3 | 0 | 0.8574 | 0.7290 | 0.6141 | 0.5120 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 |
| | 1 | 0.9928 | 0.9720 | 0.9392 | 0.8960 | 0.8438 | 0.7840 | 0.7182 | 0.6480 | 0.5748 | 0.5000 |
| | 2 | 0.9999 | 0.9990 | 0.9966 | 0.9920 | 0.9844 | 0.9730 | 0.9571 | 0.9360 | 0.9089 | 0.8750 |
| 4 | 0 | 0.8145 | 0.6561 | 0.5220 | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 |
| | 1 | 0.9860 | 0.9477 | 0.8905 | 0.8192 | 0.7383 | 0.6517 | 0.5630 | 0.4752 | 0.3910 | 0.3125 |
| | 2 | 0.9995 | 0.9963 | 0.9830 | 0.9728 | 0.9492 | 0.9163 | 0.8735 | 0.8208 | 0.7585 | 0.6875 |
| | 3 | 1.0000 | 0.9999 | 0.9995 | 0.9984 | 0.9961 | 0.9919 | 0.9850 | 0.9744 | 0.9590 | 0.9375 |
| 5 | 0 | 0.7738 | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0312 |
| | 1 | 0.9774 | 0.9185 | 0.8352 | 0.7373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 |
| | 2 | 0.9988 | 0.9914 | 0.9734 | 0.9421 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 |
| | 3 | 1.0000 | 0.9995 | 0.0078 | 0.9933 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 |
| | 4 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 |
| 6 | 0 | 0.7351 | 0.5314 | 0.3771 | 0.2621 | 0.1780 | 0.1176 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
| | 1 | 0.9672 | 0.8857 | 0.7765 | 0.6554 | 0.5339 | 0.4202 | 0.3191 | 0.2333 | 0.1636 | 0.1094 |
| | 2 | 0.9978 | 0.9842 | 0.9527 | 0.9011 | 0.8306 | 0.7443 | 0.6471 | 0.5443 | 0.4415 | 0.3438 |
| | 3 | 0.9999 | 0.9987 | 0.9941 | 0.9830 | 0.9624 | 0.9295 | 0.8826 | 0.8208 | 0.7447 | 0.6562 |
| | 4 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9954 | 0.9891 | 0.9777 | 0.9590 | 0.9308 | 0.8906 |
| | 5 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9982 | 0.9959 | 0.9917 | 0.9844 |
| 7 | 0 | 0.6983 | 0.4783 | 0.3206 | 0.2097 | 0.1335 | 0.0824 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
| | 1 | 0.9556 | 0.8503 | 0.7166 | 0.5767 | 0.4449 | 0.3294 | 0.2338 | 0.1586 | 0.1024 | 0.0625 |
| | 2 | 0.9962 | 0.9743 | 0.9262 | 0.8520 | 0.7564 | 0.6471 | 0.5323 | 0.4199 | 0.3164 | 0.2266 |

(11)--The value straight across the table from .0625 in the "x" column is "1". This is our value T(Table). See the portion of the table above. We have T(Table)=1.

(12)--T(Computed) is greater than T(Table). That is:

T(Computed)>T(Table)

2>1

We must accept the null hypothesis $H_o$: There is no

significant trend at the $\alpha=.0625$ level Our tally (Step 7) would have needed to be zero for the trend to be significant at this $\alpha$-level.

(13)--This step is not applicable, Step 12 was.

(14)--Again, .0625 is our exact level of significance.

(15)--Steps 15 through 19, choice (B), the test for upward trend, goes much the same way. Choice (C) a bit more involved, is demonstrated here.

(20)--We go to Table II and look for the 4-place decimal closest to $\alpha/2$ or $.05/2=.025$. That is, looking in the column associated with $p=.50$ and $n=7$, we find .0078 is closest to .025.

(21)--We find the value in the x-column opposite to .0078 is 0. Thus our test-statistic is T(Table)=0.

(22)--Our T(Computed)=2 0, but must check to see if it is less than n-T(Table). We find n-T(Table)=7-0=7 and T(Computed) is less than this value.

$$T(\text{Table}) < T(\text{Computed}) < n - T(\text{Table})$$

$$0<2<7$$

We must accept $H_o$: No trend.

(23)--Not applicable, Step 22 satisfied our test.

(24)--Actual exact significance-level here is 2(.0078) =.0156.

## Table II
## The Binomial Distribution
(Ref 4:477)

$$B(x;n,p) = \sum_{k=0}^{x} \binom{n}{k} p^k (1-p)^{n-k}$$

| | | p | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n | x | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| 2 | 2 | 0.9025 | 0.8100 | 0.7225 | 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 |
| | 1 | 0.9975 | 0.9900 | 0.9775 | 0.9600 | 0.9375 | 0.9100 | 0.8755 | 0.8400 | 0.7975 | 0.7500 |
| 3 | 0 | 0.8574 | 0.7290 | 0.6141 | 0.5120 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 |
| | 1 | 0.9928 | 0.9720 | 0.9392 | 0.8960 | 0.8438 | 0.7840 | 0.7182 | 0.6480 | 0.5748 | 0.5000 |
| | 2 | 0.9999 | 0.9990 | 0.9966 | 0.9920 | 0.9844 | 0.9730 | 0.9571 | 0.9360 | 0.9089 | 0.8750 |
| 4 | 0 | 0.8145 | 0.6561 | 0.5220 | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 |
| | 1 | 0.9860 | 0.9477 | 0.8905 | 0.8192 | 0.7383 | 0.6517 | 0.5630 | 0.4752 | 0.3910 | 0.3125 |
| | 2 | 0.9995 | 0.9963 | 0.9880 | 0.9728 | 0.9492 | 0.9163 | 0.8735 | 0.8208 | 0.7585 | 0.6875 |
| | 3 | 1.0000 | 0.9999 | 0.9995 | 0.9984 | 0.9961 | 0.9919 | 0.9850 | 0.9744 | 0.9590 | 0.9375 |
| 5 | 0 | 0.7738 | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0312 |
| | 1 | 0.9774 | 0.9185 | 0.8352 | 0.7373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 |
| | 2 | 0.9988 | 0.9914 | 0.9734 | 0.9421 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 |
| | 3 | 1.0000 | 0.9995 | 0.0078 | 0.9933 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 |
| | 4 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9950 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 |
| 6 | 0 | 0.7351 | 0.5314 | 0.3771 | 0.2621 | 0.1780 | 0.1176 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
| | 1 | 0.9672 | 0.8857 | 0.7765 | 0.6554 | 0.5339 | 0.4202 | 0.3191 | 0.2333 | 0.1636 | 0.1094 |
| | 2 | 0.9978 | 0.9842 | 0.9527 | 0.9011 | 0.8306 | 0.7443 | 0.6471 | 0.5443 | 0.4415 | 0.3438 |
| | 3 | 0.9999 | 0.9987 | 0.9941 | 0.9830 | 0.9624 | 0.9295 | 0.8826 | 0.8208 | 0.7447 | 0.6562 |
| | 4 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9954 | 0.9891 | 0.9777 | 0.9590 | 0.9308 | 0.9906 |
| | 5 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9982 | 0.9959 | 0.9917 | 0.9844 |
| 7 | 0 | 0.6983 | 0.4783 | 0.3206 | 0.2097 | 0.1335 | 0.0824 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
| | 1 | 0.9556 | 0.8503 | 0.7166 | 0.3767 | 0.4449 | 0.3294 | 0.2338 | 0.1586 | 0.1024 | 0.0625 |
| | 2 | 0.9962 | 0.9743 | 0.9262 | 0.8520 | 0.7564 | 0.6471 | 0.5323 | 0.4199 | 0.3164 | 0.2266 |
| | 3 | 0.9998 | 0.9973 | 0.9879 | 0.9667 | 0.9294 | 0.8740 | 0.8002 | 0.7102 | 0.6083 | 0.5000 |
| | 4 | 1.0000 | 0.9998 | 0.9988 | 0.9953 | 0.9871 | 0.9712 | 0.9444 | 0.9037 | 0.8471 | 0.7734 |
| | 5 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9962 | 0.9910 | 0.9812 | 0.9643 | 0.9375 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9984 | 0.9963 | 0.9922 |
| 8 | 0 | 0.6634 | 0.4305 | 0.2725 | 0.1678 | 0.1001 | 0.0576 | 0.0319 | 0.0168 | 0.0084 | 0.0039 |
| | 1 | 0.9428 | 0.8131 | 0.6572 | 0.5033 | 0.3671 | 0.2553 | 0.1691 | 0.1064 | 0.0632 | 0.0352 |
| | 2 | 0.9942 | 0.9619 | 0.8948 | 0.7969 | 0.6785 | 0.5518 | 0.4278 | 0.3154 | 0.2201 | 0.1445 |
| | 3 | 0.9996 | 0.9950 | 0.9786 | 0.9437 | 0.8862 | 0.8059 | 0.7064 | 0.5941 | 0.4770 | 0.3633 |
| | 4 | 1.0000 | 0.9996 | 0.9971 | 0.9896 | 0.9727 | 0.9420 | 0.8939 | 0.8263 | 0.7396 | 0.6367 |
| | 5 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9958 | 0.9887 | 0.9747 | 0.9502 | 0.9115 | 0.8555 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9964 | 0.9915 | 0.9819 | 0.9648 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9983 | 0.9961 |
| 9 | 0 | 0.6302 | 0.3874 | 0.2316 | 0.1342 | 0.0751 | 0.0404 | 0.0207 | 0.0101 | 0.0046 | 0.0020 |
| | 1 | 0.9288 | 0.7748 | 0.5995 | 0.4362 | 0.3003 | 0.1960 | 0.1211 | 0.0705 | 0.0385 | 0.0195 |
| | 2 | 0.9916 | 0.9470 | 0.8591 | 0.7382 | 0.6007 | 0.4628 | 0.3373 | 0.2318 | 0.1495 | 0.0898 |
| | 3 | 0.9994 | 0.9917 | 0.9661 | 0.9144 | 0.8343 | 0.7297 | 0.6089 | 0.4826 | 0.3614 | 0.2539 |
| | 4 | 1.0000 | 0.9991 | 0.9944 | 0.9804 | 0.9511 | 0.9012 | 0.8283 | 0.7334 | 0.6214 | 0.5000 |
| | 5 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9900 | 0.9747 | 0.9464 | 0.9006 | 0.8342 | 0.7461 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9957 | 0.9888 | 0.9750 | 0.9502 | 0.9102 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9986 | 0.9962 | 0.9909 | 0.9805 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9980 |

Table II (Continued)

| n | x | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | p | | | | | |
| 10 | 0 | 0.5987 | 0.3487 | 0.1969 | 0.1074 | 0.0563 | 0.0282 | 0.0135 | 0.0060 | 0.0025 | 0.0010 |
| | 1 | 0.9139 | 0.7361 | 0.5443 | 0.3758 | 0.2440 | 0.1493 | 0.0860 | 0.0464 | 0.0232 | 0.0107 |
| | 2 | 0.9885 | 0.9298 | 0.8202 | 0.6778 | 0.5256 | 0.3828 | 0.2616 | 0.1673 | 0.0996 | 0.0547 |
| | 3 | 0.9990 | 0.9872 | 0.9500 | 0.8791 | 0.7759 | 0.6496 | 0.5138 | 0.3823 | 0.2660 | 0.1719 |
| | 4 | 0.9999 | 0.9984 | 0.9901 | 0.9672 | 0.9219 | 0.8497 | 0.7515 | 0.6331 | 0.5044 | 0.3770 |
| | 5 | 1.0000 | 0.9999 | 0.9986 | 0.9936 | 0.9803 | 0.9527 | 0.9051 | 0.8338 | 0.7384 | 0.6230 |
| | 6 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9965 | 0.9894 | 0.9740 | 0.9452 | 0.8980 | 0.8281 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9952 | 0.9877 | 0.9726 | 0.9453 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9983 | 0.9955 | 0.9893 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 |
| 11 | 0 | 0.5688 | 0.3138 | 0.1673 | 0.0859 | 0.0422 | 0.0198 | 0.0088 | 0.0036 | 0.0014 | 0.0005 |
| | 1 | 0.8981 | 0.6974 | 0.4922 | 0.3221 | 0.1971 | 0.1130 | 0.0606 | 0.0302 | 0.0139 | 0.0059 |
| | 2 | 0.9848 | 0.9104 | 0.7788 | 0.6174 | 0.4552 | 0.3127 | 0.2001 | 0.1189 | 0.0652 | 0.0327 |
| | 3 | 0.9984 | 0.9815 | 0.9306 | 0.8389 | 0.7133 | 0.5696 | 0.4256 | 0.2963 | 0.1911 | 0.1133 |
| | 4 | 0.9999 | 0.9972 | 0.9841 | 0.9496 | 0.8854 | 0.7897 | 0.6683 | 0.5328 | 0.3971 | 0.2744 |
| | 5 | 1.0000 | 0.9997 | 0.9973 | 0.9883 | 0.9657 | 0.9218 | 0.8513 | 0.7535 | 0.6331 | 0.5000 |
| | 6 | 1.0000 | 1.0000 | 0.9997 | 0.9980 | 0.9924 | 0.9784 | 0.9499 | 0.9006 | 0.8262 | 0.7256 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9957 | 0.9878 | 0.9707 | 0.9390 | 0.8867 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9980 | 0.9941 | 0.9852 | 0.9673 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9993 | 0.9978 | 0.9941 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9995 |
| 12 | 0 | 0.5404 | 0.2824 | 0.1422 | 0.0687 | 0.0317 | 0.0138 | 0.0057 | 0.0022 | 0.0008 | 0.0002 |
| | 1 | 0.8816 | 0.6590 | 0.4435 | 0.2749 | 0.1584 | 0.0850 | 0.0424 | 0.0196 | 0.0083 | 0.0032 |
| | 2 | 0.9804 | 0.8891 | 0.7358 | 0.5583 | 0.3907 | 0.2528 | 0.1513 | 0.0834 | 0.0421 | 0.0193 |
| | 3 | 0.9978 | 0.9744 | 0.9078 | 0.7946 | 0.6488 | 0.4925 | 0.3467 | 0.2253 | 0.1345 | 0.0730 |
| | 4 | 0.9998 | 0.9957 | 0.9761 | 0.9274 | 0.8424 | 0.7237 | 0.5833 | 0.4382 | 0.3044 | 0.1938 |
| | 5 | 1.0000 | 0.9995 | 0.9954 | 0.9806 | 0.9456 | 0.8822 | 0.7873 | 0.6652 | 0.5269 | 0.3872 |
| | 6 | 1.0000 | 0.9999 | 0.9993 | 0.9961 | 0.9857 | 0.9614 | 0.9154 | 0.8418 | 0.7393 | 0.6123 |
| | 7 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9972 | 0.9905 | 0.9745 | 0.9427 | 0.8883 | 0.8062 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9983 | 0.9944 | 0.9847 | 0.9644 | 0.9270 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9992 | 0.9972 | 0.9921 | 0.9807 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9968 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 13 | 0 | 0.5133 | 0.2542 | 0.1209 | 0.0550 | 0.0238 | 0.0097 | 0.0037 | 0.0013 | 0.0004 | 0.0001 |
| | 1 | 0.8646 | 0.6213 | 0.3983 | 0.2336 | 0.1267 | 0.0637 | 0.0296 | 0.0126 | 0.0049 | 0.0017 |
| | 2 | 0.9755 | 0.8661 | 0.6920 | 0.5017 | 0.3326 | 0.2025 | 0.1132 | 0.0579 | 0.0269 | 0.0112 |
| | 3 | 0.9969 | 0.9658 | 0.8820 | 0.7437 | 0.5843 | 0.4206 | 0.2783 | 0.1686 | 0.0929 | 0.0461 |
| | 4 | 0.9997 | 0.9935 | 0.9658 | 0.9009 | 0.7940 | 0.6543 | 0.5005 | 0.3530 | 0.2279 | 0.1334 |
| | 5 | 1.0000 | 0.9991 | 0.9925 | 0.9700 | 0.9198 | 0.8346 | 0.7159 | 0.5744 | 0.4268 | 0.2905 |
| | 6 | 1.0000 | 0.9999 | 0.9987 | 0.9930 | 0.9757 | 0.9376 | 0.8705 | 0.7712 | 0.6437 | 0.5000 |
| | 7 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9944 | 0.9818 | 0.9538 | 0.9023 | 0.8212 | 0.7095 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9990 | 0.9960 | 0.9874 | 0.9679 | 0.9302 | 0.8666 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9975 | 0.9922 | 0.9797 | 0.9539 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9987 | 0.9959 | 0.9888 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9983 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 14 | 0 | 0.4877 | 0.2288 | 0.1028 | 0.0440 | 0.0178 | 0.0068 | 0.0024 | 0.0008 | 0.0002 | 0.0001 |
| | 1 | 0.8470 | 0.5846 | 0.3567 | 0.1979 | 0.1010 | 0.0475 | 0.0205 | 0.0081 | 0.0029 | 0.0009 |

## Table II (Continued)

| $n$ | $x$ | $p$ = 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 2 | 0.9699 | 0.8416 | 0.6479 | 0.4481 | 0.2811 | 0.1608 | 0.0839 | 0.0398 | 0.0170 | 0.0065 |
| | 3 | 0.9958 | 0.9559 | 0.8535 | 0.6982 | 0.5213 | 0.3552 | 0.2205 | 0.1243 | 0.0632 | 0.0287 |
| | 4 | 0.9996 | 0.9908 | 0.9533 | 0.8702 | 0.7415 | 0.5842 | 0.4227 | 0.2793 | 0.1672 | 0.0898 |
| | 5 | 1.0000 | 0.9985 | 0.9885 | 0.9561 | 0.8883 | 0.7805 | 0.6405 | 0.4859 | 0.3373 | 0.2120 |
| | 6 | 1.0000 | 0.9998 | 0.9978 | 0.9884 | 0.9617 | 0.9067 | 0.8164 | 0.6925 | 0.5461 | 0.3953 |
| | 7 | 1.0000 | 1.0000 | 0.9997 | 0.9976 | 0.9897 | 0.9685 | 0.9247 | 0.8499 | 0.7414 | 0.6074 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9978 | 0.9917 | 0.9757 | 0.9417 | 0.8811 | 0.7880 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9983 | 0.9940 | 0.9825 | 0.9574 | 0.9102 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9989 | 0.9989 | 0.9961 | 0.9886 | 0.9713 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9978 | 0.9935 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 15 | 0 | 0.4633 | 0.2059 | 0.0874 | 0.0352 | 0.0134 | 0.0047 | 0.0016 | 0.0005 | 0.0001 | 0.0000 |
| | 1 | 0.8290 | 0.5490 | 0.3186 | 0.1671 | 0.0802 | 0.0353 | 0.0142 | 0.0052 | 0.0017 | 0.0005 |
| | 2 | 0.9638 | 0.8159 | 0.6042 | 0.3980 | 0.2361 | 0.1268 | 0.0617 | 0.0271 | 0.0107 | 0.0037 |
| | 3 | 0.9945 | 0.9444 | 0.8227 | 0.6482 | 0.4613 | 0.2969 | 0.1727 | 0.0905 | 0.0424 | 0.0176 |
| | 4 | 0.9994 | 0.9873 | 0.9383 | 0.8358 | 0.6865 | 0.5155 | 0.3519 | 0.2173 | 0.1204 | 0.0592 |
| | 5 | 0.9999 | 0.9978 | 0.9832 | 0.9389 | 0.8516 | 0.7216 | 0.5643 | 0.4032 | 0.2608 | 0.1509 |
| | 6 | 1.0000 | 0.9997 | 0.9964 | 0.9819 | 0.9434 | 0.8689 | 0.7548 | 0.6098 | 0.4522 | 0.3036 |
| | 7 | 1.0000 | 1.0000 | 0.9996 | 0.9958 | 0.9827 | 0.9500 | 0.8868 | 0.7869 | 0.6535 | 0.5000 |
| | 8 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9958 | 0.9848 | 0.9578 | 0.9050 | 0.8182 | 0.6964 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9963 | 0.9876 | 0.9662 | 0.9231 | 0.8491 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9972 | 0.9907 | 0.9745 | 0.9408 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9981 | 0.9937 | 0.9824 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9963 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 16 | 0 | 0.4401 | 0.1853 | 0.0743 | 0.0281 | 0.0100 | 0.0033 | 0.0010 | 0.0003 | 0.0001 | 0.0000 |
| | 1 | 0.8108 | 0.5147 | 0.2839 | 0.1407 | 0.0635 | 0.0261 | 0.0098 | 0.0033 | 0.0010 | 0.0003 |
| | 2 | 0.9571 | 0.7892 | 0.5614 | 0.3518 | 0.1971 | 0.0994 | 0.0451 | 0.0183 | 0.0066 | 0.0021 |
| | 3 | 0.9930 | 0.9316 | 0.7899 | 0.5981 | 0.4050 | 0.2459 | 0.1339 | 0.0651 | 0.0281 | 0.0106 |
| | 4 | 0.9991 | 0.9830 | 0.9209 | 0.7982 | 0.6302 | 0.4499 | 0.2892 | 0.1666 | 0.0853 | 0.0384 |
| | 5 | 0.9999 | 0.9967 | 0.9765 | 0.9183 | 0.8103 | 0.6598 | 0.4900 | 0.3288 | 0.1976 | 0.1051 |
| | 6 | 1.0000 | 0.9995 | 0.9944 | 0.9733 | 0.9204 | 0.8247 | 0.6881 | 0.5272 | 0.3660 | 0.2272 |
| | 7 | 1.0000 | 0.9999 | 0.9989 | 0.9930 | 0.9729 | 0.9256 | 0.8406 | 0.7161 | 0.5629 | 0.4018 |
| | 8 | 1.0000 | 1.0000 | 0.9998 | 0.9985 | 0.9925 | 0.9743 | 0.9329 | 0.8577 | 0.7441 | 0.5982 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9984 | 0.9929 | 0.9771 | 0.9417 | 0.8759 | 0.7728 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9938 | 0.9809 | 0.9514 | 0.8949 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9951 | 0.9851 | 0.9616 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9991 | 0.9965 | 0.9894 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9979 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 17 | 0 | 0.4181 | 0.1668 | 0.0631 | 0.0225 | 0.0075 | 0.0023 | 0.0007 | 0.0002 | 0.0000 | 0.0000 |
| | 1 | 0.7922 | 0.4818 | 0.2525 | 0.1182 | 0.0501 | 0.0193 | 0.0067 | 0.0021 | 0.0006 | 0.0001 |
| | 2 | 0.9497 | 0.7618 | 0.5198 | 0.3096 | 0.1637 | 0.0774 | 0.0327 | 0.0123 | 0.0041 | 0.0012 |
| | 3 | 0.9912 | 0.9174 | 0.7556 | 0.5489 | 0.3530 | 0.2019 | 0.1078 | 0.0464 | 0.0184 | 0.0063 |
| | 4 | 0.9988 | 0.9779 | 0.9013 | 0.7582 | 0.5739 | 0.3887 | 0.2348 | 0.1260 | 0.0595 | 0.0245 |

## Table II (Continued)

| n | x | p = 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 17 | 5 | 0.9999 | 0.9953 | 0.9681 | 0.8943 | 0.7653 | 0.5968 | 0.4197 | 0.2639 | 0.1471 | 0.0717 |
| | 6 | 1.0000 | 0.9992 | 0.9917 | 0.9623 | 0.8929 | 0.7752 | 0.6188 | 0.4478 | 0.2902 | 0.1662 |
| | 7 | 1.0000 | 0.9999 | 0.9983 | 0.9891 | 0.9598 | 0.8954 | 0.7872 | 0.6405 | 0.4743 | 0.3145 |
| | 8 | 1.0000 | 1.0000 | 0.9997 | 0.9974 | 0.9876 | 0.5597 | 0.9006 | 0.8011 | 0.6626 | 0.5000 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9965 | 0.9873 | 0.9617 | 0.9081 | 0.8166 | 0.6855 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9968 | 0.9880 | 0.9652 | 0.9174 | 0.8338 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9970 | 0.9894 | 0.9699 | 0.9283 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9975 | 0.9914 | 0.9755 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9981 | 0.9936 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9988 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 18 | 0 | 0.3972 | 0.1501 | 0.0536 | 0.0180 | 0.0056 | 0.0016 | 0.0004 | 0.0001 | 0.0000 | 0.0000 |
| | 1 | 0.7735 | 0.4503 | 0.2241 | 0.0991 | 0.0395 | 0.0142 | 0.0046 | 0.0013 | 0.0003 | 0.0001 |
| | 2 | 0.9419 | 0.7338 | 0.4797 | 0.2713 | 0.1353 | 0.0600 | 0.0236 | 0.0082 | 0.0025 | 0.0007 |
| | 3 | 0.9891 | 0.9018 | 0.7202 | 0.5010 | 0.3057 | 0.1646 | 0.0783 | 0.0328 | 0.0120 | 0.0038 |
| | 4 | 0.9985 | 0.9718 | 0.8794 | 0.7164 | 0.5187 | 0.3327 | 0.1886 | 0.0942 | 0.0411 | 0.0154 |
| | 5 | 0.9998 | 0.9936 | 0.9581 | 0.8671 | 0.7175 | 0.5344 | 0.3550 | 0.2088 | 0.1077 | 0.0481 |
| | 6 | 1.0000 | 0.9988 | 0.9882 | 0.9487 | 0.8610 | 0.7217 | 0.5491 | 0.3743 | 0.2258 | 0.1189 |
| | 7 | 1.0000 | 0.9998 | 0.9973 | 0.9837 | 0.9431 | 0.8593 | 0.7283 | 0.5634 | 0.3915 | 0.2403 |
| | 8 | 1.0000 | 1.0000 | 0.9995 | 0.9957 | 0.9807 | 0.9404 | 0.8609 | 0.7368 | 0.5778 | 0.4073 |
| | 9 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9946 | 0.9790 | 0.9403 | 0.8653 | 0.7473 | 0.5927 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9939 | 0.9788 | 0.9424 | 0.8720 | 0.7597 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9986 | 0.9938 | 0.9797 | 0.9463 | 0.8811 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9986 | 0.9942 | 0.9817 | 0.9519 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9951 | 0.9846 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9990 | 0.9962 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 19 | 0 | 0.3774 | 0.1351 | 0.0456 | 0.0144 | 0.0042 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| | 1 | 0.7547 | 0.4203 | 0.1985 | 0.0829 | 0.0310 | 0.0104 | 0.0031 | 0.0008 | 0.0002 | 0.0000 |
| | 2 | 0.9335 | 0.7054 | 0.4413 | 0.2369 | 0.1113 | 0.0462 | [illegible] | 0.0055 | 0.0015 | 0.0004 |
| | 3 | 0.9868 | 0.8850 | 0.6841 | 0.4551 | 0.2630 | 0.1332 | 0.0591 | 0.0230 | 0.0077 | 0.0022 |
| | 4 | 0.9980 | 0.9648 | 0.8556 | 0.6733 | 0.4654 | 0.2822 | 0.1500 | 0.0696 | 0.0280 | 0.0096 |
| | 5 | 0.9998 | 0.9914 | 0.9463 | 0.8369 | 0.6678 | 0.4739 | 0.2968 | 0.1629 | 0.0777 | 0.0318 |
| | 6 | 1.0000 | 0.9983 | 0.9837 | 0.9324 | 0.8251 | 0.6655 | 0.4812 | 0.3081 | 0.1727 | 0.0835 |
| | 7 | 1.0000 | 0.9997 | 0.9959 | 0.9767 | 0.9225 | 0.5180 | 0.6656 | 0.4878 | 0.3169 | 0.1796 |
| | 8 | 1.0000 | 1.0000 | 0.9992 | 0.9933 | 0.9713 | 0.9161 | 0.8145 | 0.6675 | 0.4940 | 0.3238 |
| | 9 | 1.0000 | 1.0000 | 0.9999 | 0.9984 | 0.9911 | 0.9674 | 0.9125 | 0.8139 | 0.5710 | 0.5000 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9977 | 0.9895 | 0.9653 | 0.9115 | 0.8159 | 0.6762 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9972 | 0.9886 | 0.9648 | 0.9129 | 0.8204 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9884 | 0.9658 | 0.9165 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9969 | 0.9891 | 0.9682 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9972 | 0.9904 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9978 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 |
| | 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 20 | 0 | 0.3585 | 0.1216 | 0.0388 | 0.0115 | 0.0032 | 0.0003 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
| | 1 | 0.7358 | 0.3917 | 0.1756 | 0.0692 | 0.0243 | 0.0076 | 0.0021 | 0.0005 | 0.0001 | 0.0000 |
| | 2 | 0.9245 | 0.6769 | 0.4049 | 0.2061 | 0.0913 | 0.0355 | 0.0121 | 0.0036 | 0.0009 | 0.0002 |
| | 3 | 0.9841 | 0.8670 | 0.6477 | 0.4114 | 0.2252 | 0.1071 | 0.0444 | 0.0160 | 0.0049 | 0.0013 |
| | 4 | 0.9974 | 0.9568 | 0.8298 | 0.6296 | 0.4148 | 0.2375 | 0.1182 | 0.0510 | 0.0189 | 0.0059 |
| | 5 | 0.9997 | 0.9887 | 0.9327 | 0.8042 | 0.6172 | 0.4164 | 0.2454 | 0.1256 | 0.0553 | 0.0207 |
| | 6 | 1.0000 | 0.9976 | 0.9781 | 0.9133 | 0.7858 | 0.6080 | 0.4166 | 0.2500 | 0.1299 | 0.0577 |
| | 7 | 1.0000 | 0.9996 | 0.9941 | 0.9679 | 0.8982 | 0.7723 | 0.6010 | 0.4159 | 0.2520 | 0.1316 |
| | 8 | 1.0000 | 0.9999 | 0.9987 | 0.9900 | 0.9591 | 0.8867 | 0.7624 | 0.5956 | 0.4143 | 0.2517 |
| | 9 | 1.0000 | 1.0000 | 0.9998 | 0.9974 | 0.9861 | 0.9520 | 0.8782 | 0.7553 | 0.5914 | 0.4119 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9994 | 0.9961 | 0.9829 | 0.9468 | 0.8725 | 0.7507 | 0.5881 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9949 | 0.9804 | 0.9435 | 0.8692 | 0.7483 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9587 | 0.9940 | 0.9790 | 0.9420 | 0.8684 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9985 | 0.9935 | 0.9786 | 0.9423 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9936 | 0.9793 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9985 | 0.9941 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 |
| | 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 |
| | 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

GUIDESHEET #12. Kendall's Tau Test for Trend.

Assumptions:

1---This method is used when the assumptions required for Control-Chart significance testing are not valid assumptions. Also, this method allows quicker significance-testing in most cases, though it is not as powerful a test.

Steps:

1---Arrange the data in vertical columns (as in example) with column headings as follows. Columns 4, 5, and 6 will be filled in later.

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Rank of $y_i$ | Ranks Greater G | Ranks Less L |
|---|---|---|---|---|---|
| | | | | | |

2---Count the total number of observations or measurements that are to be tested for trend. Call this number "n".

3---Rank the measurements by size. That is, the smallest number in column 3 will receive the rank of "1" and the largest the rank of "n" (if no ties), whatever "n" was in Step 2. Record each rank in column 4, Rank of $y_i$. If there are tied measurements, both (all) of them should be given the average of the two (all) successive ranks they would have received if one were bigger than the other.

4---Consider the top entry in column 4, the rank of $y_1$. Count the number of ranks which exceed it. Record this number at the top of the "G" column, column 5.

5---Consider the next entry in column 4, the rank of $y_2$. Count the number of remaining ranks in column 4

which exceed it. That is, the rank of $y_1$ is not to be considered again. Record this number in the appropriate place in column 5.

6 ---Continue counting exceeding ranks for remaining measurements, filling column 5.

7 ---Compute the sum of the values in column 5 ($\Sigma G$) and record this sum below column 5.

8 ---Again, consider the top entry in column 4, the rank of $y_1$. Count the number of ranks which are less than it. Record this number at the top of the "L" column, column 6.

9 ---Consider the next entry in column 4 (rank of $y_2$). Count the remaining ranks which are smaller and record the number in column 6.

10 --Continue for the remaining entries of column 4, filling column 6.

11 --Compute the sum of the values in column 6 ($\Sigma L$) and record this sum below column 6.

12 --Compute the value of the test-statistic, T(Computed).

$$T(\text{Computed}) = \sum G - \sum L$$

13 --Choose a significance level $\alpha$. A significance level of $\alpha$=.05 or $\alpha$=.10 is usually appropriate.

14 --Form the null hypothesis $H_o$ and alternative hypothesis $H_1$. These may be picked from the three choices below.

(A) $H_o$: There is no downward trend.
$H_1$: There is a downward trend in the data.

(B) $H_o$: There is no upward trend.
$H_1$: There is an upward trend in the data.

(C) $H_o$: No trend exists.
$H_1$: There is either an upward trend of a downward trend in the data.

If (A) is chosen, continue with Step 15. If (B) is chosen, go to Step 17. If (C) is chosen, go to Step 19.

15 --Enter Table III with $p=1-\alpha$ (Step 13) and the value of n (Step 2) to find the numerical value of the test-statistic. When testing for a downward trend, T(Table) equals the negative of the value found in the $p=1-\alpha$ column across from the proper value of n.

16 --If T(Computed) from Step 12 is greater than or equal to T(Table), T(Computed)$\geq$T(Table), accept the null hypothesis $H_o$ of no downward trend. If T(Computed) <T(Table), reject the null hypothesis in favor of the alternative hypothesis $H_1$: There is a downward trend in the data. This step completes the test for a downward trend.

---

17 --Enter Table III with $p=1-\alpha$ (Step 13) and the value of n (Step 2) to find the table value of the test-statistic T(Table). That is, T(Table) is found in the $p=1-\alpha$ column across from the proper value of n.

18 --If T(Computed) from Step 12 is less than or equal to T(Table), T(Computed)$\leq$T(Table), accept the null hypothesis $H_o$ of no upward trend. If T(Computed) >T(Table), reject the null hypothesis in favor of the alternative hypothesis $H_1$: There is an upward trend in the data. This step completes the test for an upward trend.

---

19 --Enter Table III with $p=1-\alpha/2$ ($\alpha$ from Step 13) and the value of n (Step 2) to find the table value of the test-statistic T(Table). That is, T(Table) is found in the $p=1-\alpha/2$ column across from the proper value of n.

(20) --If T(Computed) from Step 12 is less than the <u>negative</u> of T(Table) <u>or</u> greater than T(Table), <u>reject</u> the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is either a downward trend or an upward trend in the data. That is, if T(Computed) $<-$T(Table) <u>or</u> T(Computed)$>$T(Table), reject $H_o$ in favor of $H_1$.

(21) --If T(Computed) from Step 12 is greater than or equal to the <u>negative</u> of T(Table) and less than or equal to T(Table), accept the null hypothesis $H_o$ of no trend. That is, if $-$T(Table)$\leq$T(Computed)$\leq$T(Table), accept $H_o$.

EXAMPLE for Kendall's Tau Test for Trend (Guidesheet #12)

Steps:

1---The data is arranged as follows:

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Rank of $y_i$ | Ranks Greater G | Ranks Less L |
|---|---|---|---|---|---|
| Jan | 1 | 89.9 | 2 | 12 | 0 |
| Feb | 2 | 90.5 | 5.5 | 9 | 3 |
| Mar | 3 | 92.2 | 15 | 0 | 12 |
| Apr | 4 | 91.2 | 11 | 3 | 8 |
| May | 5 | 90.7 | 7.5 | 5 | 4 |
| Jun | 6 | 91.3 | 12 | 2 | 7 |
| Jul | 7 | 90.7 | 7.5 | 4 | 4 |
| Sep | 8 | 91.7 | 14 | 0 | 7 |
| Oct | 9 | 89.9 | 2 | 5 | 0 |
| Nov | 10 | 90.1 | 4 | 4 | 1 |
| Dec | 11 | 90.8 | 9 | 2 | 2 |
| Jan | 12 | 90.5 | 5.5 | 2 | 1 |
| Feb | 13 | 91.4 | 13 | 0 | 2 |
| Mar | 14 | 89.9 | 2 | 1 | 0 |
| Apr | 15 | 91.1 | 10 | 0 | 0 |

$\Sigma G = \underline{49}$ $\Sigma L = \underline{51}$

2---Use all the data, giving us n=15.

3---The value 89.9 is the smallest but this value appears 3 times in data. If these were not tied, but just barely unequal, their ranks would be 1, 2, 3. The average of these is $\frac{1+2+3}{3}=2$. The rank of "2" is assigned to all three. The value 90.1 is ranked "4", both 90.5's ranked 5.5.

4---Consider the rank of $y_1$, the number "2". The number 2 is exceeded by a total of 12 numbers underneath it in the column. Record "12" in column #5.

5---Consider the next rank, the rank of $y_2$ which is "5.5". Nine (9) numbers underneath "5.5" exceed the number "5.5". Record "9". Next, <u>no</u> ranks exceed 15, record "0". Next, 3 ranks exceed 8 (remember only ranks

underneath are considered) so record "3".

6---Continue as in Step 5, filling column #5 as shown.

7---If we add up column #5 we get $\Sigma G=49$. Record this in the space designated above.

8---Consider again the rank of $y_1$, the number "2". There are zero ranks underneath the number 2 which are less. Record "0" in column #6.

9---Consider the rank of $y_2$, the number 5.5. There are 3 smaller ranks underneath the value 5.5 in column #4. Record "3" in column #6.

10--Consider the remaining ranks of column #4. For rank "12" we get "12", for rank "8" we get "8", for rank "5.5" we get "4" and so on, filling column #6 as shown.

11--Add up the numbers in column #6. We get $\Sigma L=51$ and record below column #6.

12--$T(\text{Computed}) = \sum G - \sum L = 49-51 = -2$

$T(\text{Computed}) = -2$

13--We choose $\alpha=.05$

14--We are mostly interested in the presence of a downward trend, so we choose (A).

15--Go to Table III, page 134. $p=1-\alpha=1-.05=.950$ and $n=15$. Go down the $p=.950$ column and across from $n=15$ we find "33". The negative of this is our Table value, so

$T(\text{Table}) = \underline{-33}$

| *n* | *p* = .900 | .950 |
|---|---|---|
| 4 | 4 | 4 |
| 5 | 6 | 6 |
| 6 | 7 | 9 |
| 7 | 9 | 11 |
| 8 | 10 | 14 |
| 9 | 12 | 16 |
| 10 | 15 | 19 |
| 11 | 17 | 21 |
| 12 | 18 | 24 |
| 13 | 22 | 26 |
| 14 | 23 | 31 |
| 15 | 27 | 33 |
| [illegible] | [illegible] | [illegible] |

16--The value -2 is greater than -33 (less negative means greater) so we accept $H_o$: No significant downward

trend at the .05 significance level. That is
-2>-33, accept $H_o$.

---

The choice (B) for test of upward trend is much the same. Let's say we chose (C).

(19) --Go to Table III, page 134. p=1- /2=1-.05/2=1-.025 =.975. Look under the p=.975 column and across from n=15 and find "39". The value.

(20) --T(Computed)=-2 is <u>neither</u> less than the negative of T(Table)=-39 <u>nor</u> greater than T(Table)=39, so we <u>cannot</u> reject $H_o$. Go to Step 21.

(21) --We find -39≤-2≤39. That is, -T(Table)≤T(Computed) ≤T(Table) so we <u>accept</u> the null hypothesis $H_o$: No significant trent exists at the .05 level of significance.

## Table III

### Quantiles of the Kendall Test Statistic

(Ref 1:391,392)

| $n$ | $p = .900$ | .950 | .975 | .990 | .995 |
|---|---|---|---|---|---|
| 4 | 4 | 4 | 6 | 6 | 6 |
| 5 | 6 | 6 | 8 | 8 | 10 |
| 6 | 7 | 9 | 11 | 11 | 13 |
| 7 | 9 | 11 | 13 | 15 | 17 |
| 8 | 10 | 14 | 16 | 18 | 20 |
| 9 | 12 | 16 | 18 | 22 | 24 |
| 10 | 15 | 19 | 21 | 25 | 27 |
| 11 | 17 | 21 | [illegible] | 29 | 31 |
| 12 | 18 | 24 | 28 | 34 | 36 |
| 13 | 22 | 26 | 32 | 38 | 42 |
| 14 | 23 | 31 | 35 | 41 | 45 |
| 15 | 27 | 33 | 39 | 47 | 51 |
| 16 | 28 | 36 | 44 | 50 | 56 |
| 17 | 32 | 40 | 48 | 56 | 62 |
| 18 | 35 | 43 | 51 | 61 | 67 |
| 19 | 37 | 47 | 55 | 65 | 73 |
| 20 | 40 | 50 | 60 | 70 | 78 |
| 21 | 42 | 54 | 64 | 76 | 84 |
| 22 | 45 | 59 | 69 | 81 | 89 |
| 23 | 49 | 63 | 73 | 87 | 97 |
| 24 | 52 | 66 | 78 | 92 | 102 |
| 25 | 56 | 70 | 84 | 98 | 108 |
| 26 | 59 | 75 | 89 | 105 | 115 |
| 27 | 61 | 79 | 93 | 111 | 123 |
| 28 | 66 | 84 | 98 | 116 | 128 |
| 29 | 68 | 88 | 104 | 124 | 136 |
| 30 | 73 | 93 | 109 | 129 | 143 |
| 31 | 75 | 97 | 115 | 135 | 149 |
| 32 | 80 | 102 | 120 | 142 | 158 |
| 33 | 84 | 106 | 126 | 150 | 164 |
| 34 | 87 | 111 | 131 | 155 | 173 |
| 35 | 91 | 115 | 137 | 163 | 179 |
| 36 | 94 | 120 | 144 | 170 | 188 |
| 37 | 98 | 126 | 150 | 176 | 196 |
| 38 | 103 | 131 | 155 | 183 | 203 |
| 39 | 107 | 137 | 161 | 191 | 211 |
| 40 | 110 | 142 | 168 | 198 | 220 |

GUIDESHEET #12. Spearman's Test for Trend

Assumptions:

(1)---This method is used when the assumptions required for Control-Chart significance-testing a not valid assumptions. Also, this method allows <u>quicker</u> significance-testing in most cases, though it is not as powerful a test.

Steps:

(1)---Arrange the data in vertical columns (as in example) with column headings as follows. Columns 4 through 7 will be filled in later.

| Period Name | Period Number $x_i$ | Meas-ure-ment $y_i$ | Rank of $x_i$ $R(x_i)$ | Rank of $y_i$ $R(y_i)$ | Difference $R(x_i)-R(y_i)$ | Difference Squared $(R(x_i)-R(y_i))^2$ |
|---|---|---|---|---|---|---|
| | | | | | | |

(2)---Count the total number of observations or measurements that are to be tested for trend. Call this number "n".

(3)---This method uses the ranks of both the time periods and the measurements. Since the time periods are recorded in their natural order, the period numbers are naturally ranked in size from smallest to largest. Therefore, for column 4, Ranks of $x_i$, just duplicate the entries of column 2.

(4)---Rank the measurements by size. That is, the smallest number in column 3 will receive the rank of "1" and the largest the rank of "n", whatever "n" was in Step 2. Record each rank in column 5, Rank of $y_i$. If there are tied measurements, both (all) of them should be given the <u>average</u> of the two (all)

successive ranks they would have received if one were bigger than the other.

5---For each measurement, subtract the ranks and enter this difference (whether positive or negative) in column 6, $R(x_i)-R(y_i)$.

6---Square each entry in column 6 and enter each number in column 7, $\left(R(x_i)-R(y_i)\right)^2$ .

7---Add up the values in column 7 and record this sum, $\Sigma\left(R(x_i)-R(y_i)\right)^2$ , below column 7 This is the computed value of the test-statistic T(Computed).

8---Choose a significance level $\alpha$. A significance level of $\alpha=.05$ or $\alpha=.10$ is usually appropriate.

9---Form the null hypothesis $H_o$ and alternative hypothesis $H_1$. These may be picked from the three choices below.

(A) $H_o$: There is no downward trend.
$H_1$: There is a significant downward trend.

(B) $H_o$: There is no upward trend.
$H_1$: There is a significant upward trend in the data.

(C) $H_o$: There is no significant trend.
$H_1$: There is either an upward trend or a downward trend in the data.

If (A) is chosen, continue with Step 10. If (B) is chosen, go to Step 13. If (C) is chosen, go to Step 16.

10--Enter Table IV, page 142, with $p=\alpha$, chosen in Step 8, and the "n" from Step 2 to find the table value of the test-statistic. That is, T(Table) is found in the p column across from the proper value of n. Also, the value $1/3\ n(n^2-1)$ will be needed for the test. It may be computed, but is provided in the far-right column across from the proper value of n.

(11) --If T(Computed) is less than or equal to $1/3n(n^2-1)$ minus T(Table), accept the null hypothesis $H_o$: There is no downward trend. That is, if the following is true, accept $H_o$ at the $\alpha$-level of significance.

$$T(\text{Computed}) \leq \frac{1}{3} n(n^2-1) - T(\text{Table})$$

(12) --If T(Computed) is greater than $1/3n(n^2-1)$ minus T(Table), reject the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is a significant downward trend. That is, if the following is true, accept $H_1$.

$$T(\text{Computed}) > \frac{1}{3} n(n^2-1) - T(\text{Table})$$

This completes the test for a downward trend.

---

(13) --Enter Table IV, page 142, with $p=\alpha$, chosen in Step 8, and the "n" from Step 2 to find the table value of the test-statistic. That is, T(Table) is found in the $p=\alpha$ column across from the proper value of n.

(14) --If T(Computed) from Step 7 is greater than or equal to T(Table), accept the null hypothesis $H_o$: There is no upward trend. That is, if $T(\text{Computed}) \geq T(\text{Table})$, accept $H_o$.

(15) --If T(Computed) is less than T(Table), reject the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is an upward trend. That is, if $T(\text{Computed}) < T(\text{Table})$, accept $H_1$. This completes the test for an upward trend.

---

(16) --Enter Table IV, page 142, with $p=\alpha/2$, chosen in Step 8, and the n from Step 2 to find the table value of the test-statistic T(Table). That is,

T(Table) is found in the $p=\alpha/2$ column across from the proper value of n.

17 --If T(Computed) from Step 7 is greater than or equal to T(Table) <u>and</u> less than or equal to $1/3n(n^2-1)$ minus T(Table), accept the null hypothesis $H_o$: There is no significant trend. That is, if the following is <u>true</u>, accept the null hypothesis $H_o$.

$$T(\text{Table}) \leq T(\text{Computed}) \leq \frac{1}{3} n(n^2-1) - T(\text{Table})$$

18 --If T(Computed) is less than T(Table) <u>or</u> greater than $1/3n(n^2-1)$-T(Table), reject the null hypothesis $H_o$ in favor of the alternative hypothesis $H_1$: There is either an upward or downward trend. That is, if <u>either one</u> of the following relationships is true, accept $H_1$.

$$T(\text{Computed}) < T(\text{Table})$$

<u>or</u> $$T(\text{Computed}) > \frac{1}{3} n(n^2-1) - T(\text{Table})$$

This completes the Spearman Test for trend.

EXAMPLE for Spearman Test for Trend (Guidesheet #13)

Steps:

(1)---The data is arranged as follows:

| Period Name | Period Number $x_i$ | Measurement $y_i$ | Rank $R(x_i)$ | Rank $R(y_i)$ | Difference $R(x_i)-R(y_i)$ | Difference Squared $(R(x_i)-R(y_i))^2$ |
|---|---|---|---|---|---|---|
| Jan | 1 | 96.6 | 1 | 9 | -8 | 64 |
| Feb | 2 | 96.5 | 2 | 8 | -6 | 36 |
| Mar | 3 | 96.2 | 3 | 7 | -4 | 16 |
| Apr | 4 | 95.9 | 4 | 6 | -2 | 4 |
| May | 5 | 95.8 | 5 | 5 | 0 | 0 |
| Jun | 6 | 95.6 | 6 | 4 | 2 | 4 |
| Jul | 7 | 94.8 | 7 | 2.5 | 4.5 | 20.25 |
| Aug | 8 | 94.8 | 8 | 2.5 | 5.5 | 30.25 |
| Sep | 9 | 97.0 | 9 | 10 | 1 | 1 |
| Nov | 10 | 94.5 | 10 | 1 | 9 | 81 |

$\sum(R(x_i)-R(y_i))^2$

$= \underline{256.5}$

(2)---We will use all data for the test, so n=10

(3)---Rewrite the numbers in column #2 in column #4 as shown above.

(4)---The smallest value of $y_i$ is 94.2. It receives the rank of "1". The next largest value $y_8$=94.8 is tied with $y_7$=94.8. If one were bigger than the other, they would have received ranks of 2 and 3. We will give each the average rank of 2.5. Rank the remainder as is shown in the table, column #5.

(5)---Subtract the rank 9 from the rank 1. Record the difference (-8) in column #6 as shown. Do this for each pair of measurements.

(6)---The square of -8 is +64. This is recorded in column #7. The square of -6 is +36, and so on as shown.

7---Adding the numbers in column #7 gives

$$\sum(R(x_i)-R(y_i))^2 = \underline{256.5}$$

as recorded below column #7. T(computed)=256.5

8---We will use the $\alpha$=.05 level of significance.

9---We are interested primarily in the presence of a downward trend, so (A) is chosen.

10--Go to Table IV, page 142. We look in the p=.050 column and across from n=10 and find T(Table)=74. The value for 1/3n($n^2$-1)=330 (far right column)

| $n$ | $p = .001$ | .005 | .010 | .025 | .050 | .100 | $\frac{1}{3}n(n^2-1)$ |
|---|---|---|---|---|---|---|---|
| 4 | | | | | 2 | 2 | 20 |
| 5 | | | 2 | 2 | 4 | 6 | 40 |
| 6 | | 2 | 4 | 6 | 8 | 14 | 70 |
| 7 | 2 | 6 | 8 | 14 | 18 | 26 | 112 |
| 8 | 6 | 12 | 16 | 24 | 32 | 42 | 168 |
| 9 | 12 | 22 | 28 | 38 | 50 | 64 | 240 |
| 10 | 22 | 36 | 44 | 60 | 74 | 92 | 330 |

11--T(Computed)=256.5 is not less than or equal to 1/3n($n^2$-1)-T(Table) so we do not accept $H_o$. Go to Step 13. That is $256.5 \overset{?}{<} 330-74=256$ is not true.

12--T(Computed)=256.5 is greater than 1/3n($n^2$-1)-T(Table) so we reject $H_o$ in favor of $H_1$: There is a significant downward trend at $\alpha$=.05. That is, since

$$256.5>330-74 = 256$$

accept $H_1$.

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If (B) were chosen, we would not need the value of 1/3n($n^2$-1). The value T(Computed)=256.5 would be tested, against T(Table)=256. Since 256.5>256, the null hypothesis ($H_o$: No upward trend) would be accepted.

16 - 18--

If (C) were chosen, we would have to use the p=$\alpha$/2

$=.05/2=.025$ column across from $n=10$ to get T(Table) $=60$. We must also use $1/3n(n^2-1)=330$ from the far right column.

Is $T(\text{Table}) \le T(\text{Computed}) \le \frac{1}{3}n(n^2-1) - T(\text{Table})$ true?

We have: $60 \overset{?}{\le} 256.5 \overset{?}{\le} 330-60 = 270$

Yes, this is true.

We must now accept $H_o$: There is no significant trend at the $\alpha=.05$ level. When testing for both kinds of trend, the data must exhibit a more severe trend before the null hypothesis is rejected, unlike when testing for one kind of trend (Step 12).

Table IV

Critical Values for Spearman Test
(Ref 1:389)

| $n$ | $p = .001$ | .005 | .010 | .025 | .050 | .100 | $\frac{1}{3}n(n^2 - 1)$ |
|---|---|---|---|---|---|---|---|
| 4 | | | | | 2 | 2 | 20 |
| 5 | | | 2 | 2 | 4 | 6 | 40 |
| 6 | | 2 | 4 | 6 | 8 | 14 | 70 |
| 7 | 2 | 6 | 8 | 14 | 18 | 26 | 112 |
| 8 | 6 | 12 | 16 | 24 | 32 | 42 | 168 |
| 9 | 12 | 22 | 28 | 38 | 50 | 64 | 240 |
| 10 | 22 | 36 | 44 | 60 | 74 | 92 | 330 |
| 11 | 36 | 56 | 66 | 86 | 104 | 128 | 440 |
| 12 | 52 | 78 | 94 | 120 | 144 | 172 | 572 |
| 13 | 76 | 110 | 130 | 162 | 190 | 226 | 728 |
| 14 | 106 | 148 | 172 | 212 | 246 | 290 | 910 |
| 15 | 142 | 194 | 224 | 270 | 312 | 364 | 1120 |
| 16 | 186 | 250 | 284 | 340 | 390 | 450 | 1360 |
| 17 | 238 | 314 | 356 | 420 | 480 | 550 | 1632 |
| 18 | 304 | 390 | 438 | 512 | 582 | 664 | 1938 |
| 19 | 372 | 476 | 532 | 618 | 696 | 790 | 2280 |
| 20 | 454 | 574 | 638 | 738 | 826 | 934 | 2660 |
| 21 | 546 | 686 | 758 | 870 | 972 | 1092 | 3080 |
| 22 | 652 | 810 | 892 | 1020 | 1134 | 1270 | 3542 |
| 23 | 772 | 950 | 1042 | 1184 | 1312 | 1464 | 4048 |
| 24 | 904 | 1104 | 1208 | 1366 | 1510 | 1678 | 4600 |
| 25 | 1050 | 1274 | 1390 | 1566 | 1726 | 1912 | 5200 |
| 26 | 1212 | 1462 | 1590 | 1786 | 1960 | 2168 | 5850 |
| 27 | 1390 | 1666 | 1808 | 2024 | 2216 | 2444 | 6552 |
| 28 | 1586 | 1890 | 2046 | 2284 | 2494 | 2744 | 7308 |
| 29 | 1800 | 2134 | 2306 | 2564 | 2796 | 3068 | 8120 |
| 30 | 2032 | 2398 | 2584 | 2868 | 3120 | 3416 | 8990 |

## V. Conclusions and Recommendations

Chapter IV may be extracted and used as a guidebook with a minimum of changes, such as the page numbers. This chapter describes the conclusions reached as a result of the research effort and offers some recommendations for the use of the guide and for further study and development.

### Conclusions

This research effort has produced a number of methods by which a trend in measurements over time may be analyzed. As mentioned in the first chapter (page 4), an early conclusion was that a guide was needed so these methods could be applied by those with little mathematical background. While the guide gives step-by-step procedures, it does not relieve the user of all subjective judgement. The variety of methods available, the several choices of significance levels, plus the choices as to how far to project trends all give the Trend Analysis (TA) officer flexibility to do what he feels most appropriate, according to experience. It is concluded, then, that if nothing else, the guide provides quantification to the decision made by the TA officer, and he can show (with numbers) the decision-maker (the Commander, usually) the effects of the choices made during trend-method application. That is, he could say that a time-plot suggested a curved-line trend, and at a .05 level of significance, method #6 (Guidesheet #6) shows a significant departure from the standard will occur

in two time-periods. If the decision-maker does not accept the curved-line, the significance-level, or the number of time-periods forecasted, he and the TA officer still have a quantifiable basis upon which to argue. Alternative spproaches would be easily discussed at this point. The TA officer's expertise (or decision-maker's) can then be applied to reversing the trend as appropriate. The guide can then be used to evaluate the effects of policy changes.

## Recommendations

While this research effort discussed eight different methods to evaluate the significance of trends and one method to evaluate correlation, time factors prevented the description of other useful methods. Alternate methods of correlation were to be described but there was no time. However, if the user goes to the body of the thesis (chapter 3), he will see at least a minimal description of a non-parametric test for correlation. It uses the Cox and Stuart test for trend and refers to a bibliographical reference for a more complete discussion. Incidentally, that reference may be reasonably well understood by the non-mathematician.

Another recommendation for further development is the use of the Cox and Stuart test for trend (Guidesheet #11) for cycling data. The writer first wished to address methods that could be used when there were a limited number of data points. Usually cycling data does not show up until

measurements extend over at least a couple years. If is recommended, then, that the problem of cycling data be addressed, either by the Cox and Stuart method or others. As a start, cycling data problems are addressed in Reference 1, page 134. Also, other non-parametric correlation tests are discussed in this reference.

If this guide is well received, it may be quite valuable to extend the regression method (Guidesheet #5) to multiple linear regression analysis. This could be applied to such problems as determining what factors affect bombing CEPs, for instance. One interviewee actually mentioned that he wished he had a method by which he could evaluate what had the greatest impact on mission success: pilot proficiency, navigator proficiency, weather, equipment malfunction, or aircraft flown. Multiple linear regression techniques are ideally suited to investigate such issues.

Some of the statistical packages available for computer use may be a rich area for development of additional guides. As computer access becomes easier, a TA officer may be able to apply rather sophisticated tests like those available in SPSS (Statistical Package for the Social Sciences) if he had a step-by-step guide to help him.

These recommendations conclude the presentation and discussion of this research effort. The objectives have been achieved, and the writer has gained a better feel for many statistical methods plus an appreciation for the problems a TA officer has. It is hoped the effort put into this thesis will prove useful to others.

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2. Hoel, Paul G. Elementary Statistics. New York: John Wiley and Sons, Inc., 1960.

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6. Siegel, Sidney. NonParametric Statistics. New York: McGraw-Hill, 1955.

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<u>Bibliography B: A User's Annotated Bibliography</u>


1. Hoel, Paul G. <u>Elementary Statistics</u>. New York: John Wiley and Sons, Inc., 1960.

   This was used as a reference in this thesis. It is the author's college textbook, but is written for students whose mathematical background is limited to high-school algebra. Chapter two is about sampling and is very helpful. The chapter on testing hypotheses is also very good.

2. Huff, Darrell. <u>How to Lie with Statistics</u>. New York: W. W. Norton and Co., Inc., 1954.

   This is a very interesting book with a sense of humor. It is <u>very</u> non-mathematical.

3. Kafka, F. J. D. <u>Statistics Without Numbers</u>. New York: J. J. Little and Ives Co., 1950.

   As the title suggests, this book is far from mathematical. It is addressed to the consumer of statistics. It tries to give a few basic statistical concepts to those who otherwise would be left without any tools for interpreting them.

4. Lovejoy, Elijah P. <u>Statistics for Math Haters</u>. New York: Harper and Row, Inc., 1975.

   Written <u>specifically</u> for those who are "scared" by math or statistics, it reads much like a programmed text. It is especially good for <u>definitions</u>.

5. Mason, Robert and Hermanson, Roger H. "Programmed Learning Aid in Business and Economic Statistics." Homewood, Illinois: Learning Systems Company, 1970.

   This is a programmed text. Many of the examples are business oriented, but treatment of the concepts is widely applicable. It includes a discussion of time-series (trend) analysis.

6. Murphy, Richard L. "Time Series Analysis: Looking for the Trend" Unpublished paper. Dept. of Special Management Techniques, School of Systems and Logistics, Air Force Institute of Technology, Wright-Patterson, AFB, Ohio, 1978.

   This is used in the Logistics School at AFIT and is fairly simply written. A lot of mathematics is not required, but it is a bit more rigourous than any of the above.

## Vita

David A. Brunstetter [redacted] He [redacted] went on to Kent State University in Kent, Ohio. He received his Bachelor of Arts degree in Mathematics in 1965. He was commissioned a Second Lieutenant upon graduation from Officer Training School in 1966, then went to navigator training.

After receiving his navigator wings he entered Electronic Warfare Officer School, where he was graduated in 1967. Entering SAC as a B-52 EWO, he spent tours at Plattsburgh, Fairchild and Castle AFBs, including 650 days temporary Southeast Asia duty. He was assigned to the Air Force Institute of Technology while an EWO instructor at Castle AFB in August, 1977.

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